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Multi-Rate Harmonic Balance Provides a New Solution for Nonlinear Simulation

By Ville Karanko and Taisto Tinttunen
AWR Corporation/APLAC Division

This new technique allows nonlinear analysis of systems with a large number of frequencies, where the use of traditional methods would have impractical computation requirements

Since the early 1980s, harmonic balance analysis has been the core tool for performing nonlinear frequency-domain simulation. Today's harmonic balance tools such as AWR's APLAC® simulator can handle designs with thousands of analysis frequencies and scale almost linearly with increases in circuit elements, nodes, and frequencies. However, when applied to large circuits that have many different signal sources, traditional harmonic balance has drawbacks that render it less effective because computational times are prohibitively long and enormous amounts of memory are required. To this end, AWR has introduced within its APLAC family of harmonic balance and time-domain simulators a technology called Multi-Rate Harmonic Balance (MRHB™). This new technology eliminates the limitations of traditional harmonic balance, dramatically increasing computational speed and reducing required computer memory when analyzing frequency-rich nonlinear systems that have multiple signal sources. It can potentially even solve entire subsystems such as a mobile phone transceiver in an acceptable amount of time.

Harmonic Balance in Perspective

By the late 1980s, harmonic balance analysis overtook SPICE as the required tool within an RF designer's simulation arsenal. Transient analysis, in the form of SPICE and similar techniques, took far too much time to reach a steady-state solution and even the simplest topologies containing distributed elements fell short on memory when convolution was required. The limitations of transient analysis are most pronounced with mixers and similar RF devices that move from one frequency range to another. In analysis, these multiple, widely separated frequencies are referred to as *tones*.

Multi-tone harmonic balance analysis truly made receiver and transmitter CAD possible. The numerical techniques used in these first harmonic balance engines incorporated direct matrix methods. They were very useful for steady-state analysis containing a few transistors, but when applied to larger nonlinear circuits, dense conversion matrices resulted in the need for lots of computer memory and simulation time. As the number of transistors in microwave circuits grew, so did the number of nonlinear elements and analysis frequencies in the harmonic balance algorithm—and simulation time and memory consumption skyrocketed.

Harmonic balance technology changed for the better when numerical analysis techniques suited for solving large nonlinear problems were put to use in the 1990s. Direct matrix techniques were augmented with iterative techniques and the naïve Newton iteration was replaced by so-called inexact Newton methods. Great advances were also made in the way nonlinear device computations were conducted, using more advanced and optimized Fast Fourier Transform (FFT) techniques.

While harmonic balance is a triumph for steady-state nonlinear analysis with distributed elements, it has a rather noticeable and significant limitation. As the number of tones (independent frequencies) increases, the number of mathematical unknowns that need to be solved grows geometrically. The geometric growth comes from the fact that in a multi-tone system, each circuit element must be solved not just at the harmonics of each tone, but also at many of their linear combinations as well. The end result is that if the user cannot provide useful constraining information to the harmonic balance analysis engine by limiting the frequency combinations per circuit element, the circuit must be analyzed for the same frequencies at every circuit node. For a typical multi-tone circuit, this means that a lot of CPU time is consumed to refine a zero.

The number of analysis frequencies can become a bottleneck when the number of tones grows above three. People working in numerical analysis know this phenomenon well and call it the Curse of Dimension. To understand "the curse," refer to Table 1, which demonstrates what happens in a moderately nonlinear simulation when the number of tones is increased. Even with the reasonable accuracy of so-called diamond truncation (the strategy of selecting frequency linear combinations), the growth in the total number of frequencies and hence the number of unknowns to be solved is almost an order of magnitude for every tone.

Number of tones	DIAMOND 5 truncation	BOX 5 truncation
1	6	6
2	31	61
3	116	666
4	341	7321
5	842	80526

Table 1 · Illustration of the Curse of Dimension.

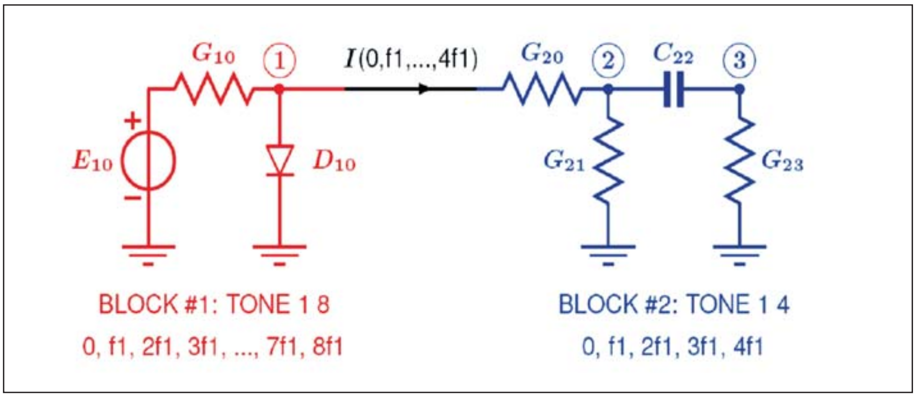

Figure 1 · This simple, three-node circuit has two blocks with different frequency settings, both of which are single tone. Block 1 has eight harmonics and Block 2 has four.

The Difference is MRHB

The core concept in MRHB is that operational blocks such as mixers, filters, and amplifiers in an RF system modify the frequency content. Traditional harmonic balance techniques assume that the relevant frequency content will be the same at every part (or block) in the circuit. In contrast, MRHB lets the designer allow different parts of the circuit to have different dominating frequencies, and that some frequencies are important to solve while others are not. This intelligent, frequency-selective technique makes it possible to solve circuits such as complex receivers with multiple stages of downconversion, multi-band power amplifiers, and complex high-frequency digital designs, an order of magnitude faster than with traditional harmonic balance. The challenge solved by MRHB is briefly summarized in "Multi-Signal Circuits: Where MRHB takes over," p. 34.

Different from traditional harmonic balance, MRHB forms its equations to solve for the multi-tone, multi-harmonic content of the circuit dynamically, adding the contribution of each element (block) only at the desired frequencies, significantly reducing the number of equations that must be solved. The analysis information is transferred from one element (block) to another via the shared frequencies as illustrated in Figure 1. This is a simple example circuit that has been divided into two blocks, each having a single-tone frequency set. The first part (red block) has eight harmonics and the second one (blue block) has four. Communication between these two circuits occurs via the five frequencies they share—DC and four harmonics. The first part is solved at all nine frequencies, which actually makes the results more accurate because of the greater number of harmonics.

For simple circuits, there is often no need or benefit to reducing the frequency content at some parts of the circuit, but many circuits require one of their nonlinear parts to be simulated accurately. With MRHB, this local accuracy requirement does not unduly affect simulation of other parts of the circuit. This means that if a frequency divider circuit requires more than 2,000 harmonics for a single-tone analysis, it can be simulated locally with a large single-tone frequency set without detrimental impact on the two-tone frequency set used in the same simulation for the mixer.

In other words, MRHB presumes that dominating frequencies differ in the various parts of a circuit and that by intelligently addressing this, a more efficient yet highly accurate harmonic balance analysis of the entire circuit can be realized. MRHB can do this while consuming less memory and less simulation time than traditional harmonic balance techniques.

Two Design Examples

To illustrate some of the fundamentals of MRHB, it helps to start with the simple circuit in Figure 1 in which the first block has a voltage source and a nonlinear diode, and the corresponding circuit equations are constructed for DC and eight harmonics. Assuming that the diode is being driven hard enough to generate significant harmonic content, an ordinary harmonic balance simulation would require the analysis of *all* circuit elements—nonlinear as well as linear—to be analyzed at all eight harmonics. The second part of the circuit implements a low-pass filter in which the higher-order harmonics would be expected to be blocked, introducing negligible, if any, signal energy at these frequencies to later elements in the circuit.

When MRHB is used as the simulation engine for analyzing this circuit's behavior, the designer is able to *individually* set the analysis of the resistors and a capacitor to have a frequency set of only DC and four harmonics. The designer can take advantage of the fact that the second block in the circuit does not exist in frequencies $5f_1$, $6f_1$, $7f_1$ and $8f_1$ and that the current I flows from Block 1 to Block 2 only for the shared frequencies, that is, DC, f_1, $2f_1$, $3f_1$ and $4f_1$. With MRHB, only the relevant frequencies and harmonics are solved on a block-by-block basis. The result is a new paradigm for nonlinear simulation as it delivers simulation accuracy more efficiently, enabling greater and greater circuit complexity to be examined.

The breakthrough capabilities of MRHB can best be understood in

Multi-Signal Circuits: Where MRHB™ Takes Over

Multi-tone systems—those with multiple input signals—create a massive simulation problem, the extent of which becomes obvious when the simulation process is examined. As a general rule of thumb, today's designers typically use single-tone harmonic balance analysis inclusive of five harmonic components (fundamental plus four additional frequencies at the first four integer multiples of the fundamental) and then look to "diamond truncation" to simplify the simulation if they add a second tone. But adding another tone is not simply tacking on one more frequency.

It is more complex than that. Beginning with a single tone, analysis is performed at DC (the fundamental frequency) and as illustrated in this example, together with the four harmonics (collectively called frequency points). This means single-tone analysis is actually performed at all the nodes in the circuit and at all of the associated frequency points. Adding another tone would perhaps at first suggest that the number of frequency points would simply be doubled from 6 to 11 or 12 (it's not necessary to solve at DC twice), however, the scaling is even worse.

In addition to solving at the second tone's fundamental and its harmonics (five in total as per the case with a single-tone), the simulator must also solve for the frequencies produced by adding and subtracting all of the various combinations of all the tones in order for accuracy to be achieved. The result is that the analysis grows the equation from 6 frequency points to 61, and this increase in complexity grows rapidly with further increases in tones/frequency points.

Now imagine simulating an amplifier that has a two-tone input signal and seven harmonics of each tone. This requires each tone to be solved at about 50 different frequencies. If four tones are required, this jumps to several thousand frequencies, and to several million frequencies for an 8-bit digital communication bus. The result is that computation very rapidly takes so long that it renders problems unsolvable in an acceptable time span and with any available memory. To simulate an entire mobile phone transceiver, the phase-locked loop alone (that includes a divider), would require 5,000 to 10,000 frequencies. Unfortunately, this makes simulation using traditional harmonic balance impractical.

MRHB was developed to accurately and rapidly simulate these types of complex problems. For example, if a design employs several filters explicitly, or areas of the circuit which induce implicit filtering, MRHB can reduce the tones and harmonics that must be considered because many of the tone/harmonic combinations or even some of the fundamental tones have no significant effect/impact on the designs performance and can safely be ignored during the simulation process.

MRHB reduces the number of frequencies that must be solved by constructing dynamic tones "on the fly" using the sums and differences of the tones defined as "sources." To analyze downconversion of an RF signal at 8 GHz with a local oscillator at 7.5 GHz, for example, their 500-MHz difference can be used as a tone for all analysis after the mixer and subsequent filtering. This intelligent block-by-block approach to harmonic balance analysis reduces problem size by a significant factor. In short, AWR's MRHB technology can produce orders of magnitude reductions in both simulation time and memory consumption over that of traditional harmonic balance.

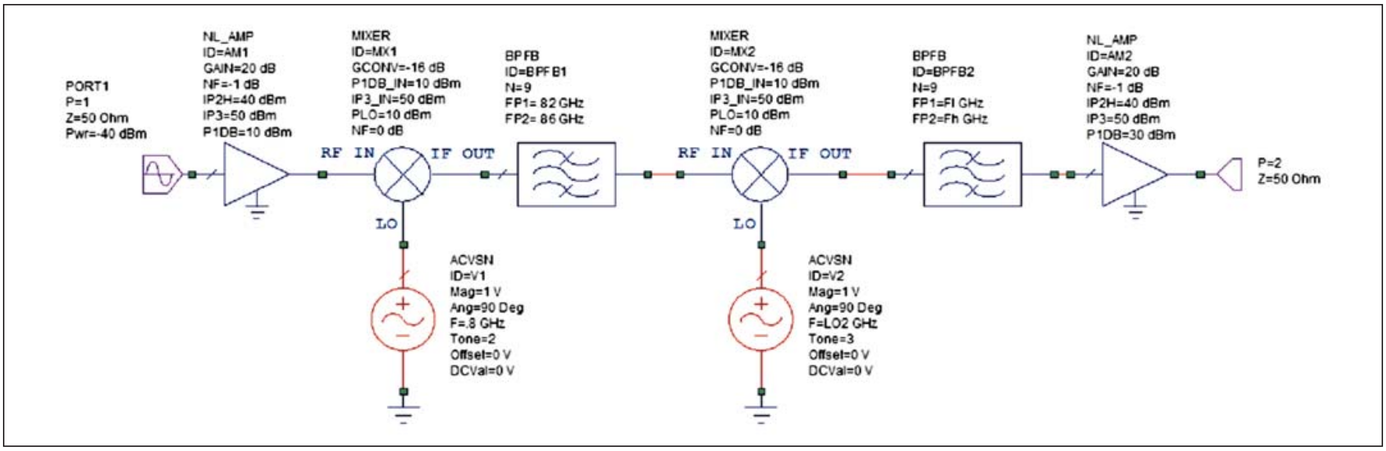

Figure 2 · A QPSK receiver that has 130 transistors and more than 100 passive components. The element with single-tone frequency settings are shown within the dashed lines and the remainder belong to a block with a two-tone frequency set.

terms of the topologies for which MRHB can be employed:

- *Single-tone, multiple-frequency domain (Figure 1):* No need for the full range of nonlinear harmonics propagated to all circuit elements because MRHB selects only the ones required.

- *Multiple-tone, multiple-frequency domain (Figure 2):* MRHB reduces the overall tone-frequency solution space yet maintains great accuracy through the use of hybrid-tones.

To illustrate this notion of hybrid tones, consider the QPSK receiver of Figure 2. This is a challenging circuit to tackle at the circuit level with harmonic balance because of its multi-tone nature and the need to have quite a few harmonics to tackle all the nonlinearities. The circuit consists of a transistor-level QPSK receiver with 130 transistors based on the BSIM3 model and more than 100 passive elements. Two-tone harmonic balance analysis would traditionally be used with a box-style truncation up to the seventh order in the RF (f_{RF} = 2.45 GHz) and the fourth order in the LO frequency (f_{LO} = 2.44 GHz). However, by partitioning the blocks based on their frequency content, MRHB analysis can employ a multi-tone frequency set and eliminates many of the harmonics in the circuit elements where they are clearly not a factor.

For the subcircuit represented by the mixer in Figure 2, the simulation results are achieved in half the time and with half the memory with no difference in measurement versus simulated results when compared with traditional harmonic balance. For the differential-to-single-ended and Bessel filter blocks, the simulation consists of a single-tone, fourth-order analysis in which the funda-

Parameter	Traditional harmonic balance	MRHB
CPU time(s)	43.9	13.5
Memory usage (Mbytes)	50	23

Table 2 · Performance comparison between harmonic balance and MRHB for Figure 2 QPSK receiver.

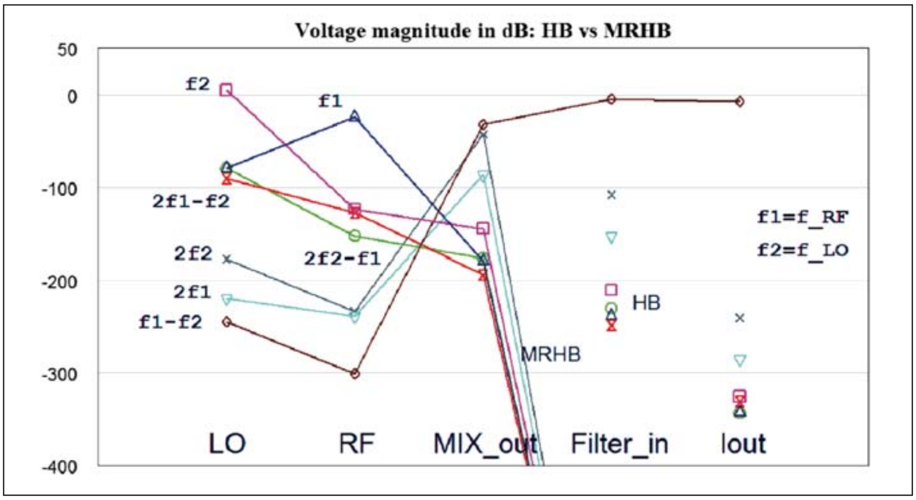

Figure 3 · The mixer subcircuit of Figure 2 output spectrum from traditional harmonic balance (△) and MRHB (☐).

mental frequency is constructed as an MRHB hybrid tone, $f_{RF}-f_{LO}$. By using this unique feature of MRHB in conjunction with the software's ability to set multi-tonal frequency analysis on a block-by-block basis, the resources necessary to analyze the receiver are reduced well beyond what is possible with traditional harmonic balance using either box or diamond truncation.

Table 2 summarizes the memory consumption and processor times of the traditional harmonic balance versus MRHB simulation for this design. Voltage magnitude is plotted in Figure 3 from different parts of the circuit at selected frequencies (LO and RF), their second harmonics, and their third-order intermodulation products. Simulated data agrees very well with the actual circuit behavior and yet was achieved in nearly one-fourth the simulation time while using half the memory.

Summary

Traditional harmonic balance analysis is struggling to keep pace with the industry's need to solve large and larger high-frequency circuits. However, the intelligent frequency selectivity and other techniques within MRHB bring new life to harmonic balance and make it ready to tackle the challenges these circuits present. AWR has applied for a patent covering MRHB, which has been in development for several years. It is now a core component of AWR's APLAC simulator portfolio and is available within AWR's Design Environment Version 2009.

Further Reading:

1. J. Virtanen, V. Karanko, T. Tinttunen, M. Heimlich, "Frequency Selective Harmonic Balance Analysis," *Proceedings of EuMC 2009,* Rome, Italy, September 2009.

2. M. Honkala, V. Karanko, J. Roos, M. Valtonen, "Frequency/Time Block Preconditioners for Harmonic Balance Jacobians," *Proceedings of ECCTD '09,* Antalya, Turkey, August 2009.

Author Information

Taisto Tinttunen is director of engineering at AWR-APLAC's division in Finland and is responsible for the company's time domain and harmonic balance technology. He joined AWR in 2005 when AWR Corp. acquired APLAC. Taisto received his MSEE at the Helsinki University of Technology.

Ville Karanko is a senior development engineer at AWR-APLAC's division in Finland and a key contributor to the company's new MRHB technology. Ville joined AWR-APLAC in 2006 and holds a Masters degree in technical mathematics and physics from the Helsinki University of Technology.