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ASK THE EXPERTS

Help Needed With Electromagnetics

Editor:

I am a graduate student in EE, and one of the things that has been hard to grasp is electromagnetics. I can work all the math, but it would be a lot easier if there was a more real-world explanation of Maxwell's equations. What I'd like to see is something similar to what I had in sophomore Mechanics class. I took the Physics Department version of this class and all the concepts of Calculus became clear after working through the same problems that Newton solved. My professor was especially good at including history notes in his lectures, which helped a lot.

Do you know of something similar for Maxwell's equations?

Michael T.
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Visualizing Maxwell's Equations

Michael, you have made a pretty tough request, but we'll try, using material drawn from Joseph White's recent book [1]. Dr. White has attempted to do just what you are looking for in his chapter on EM.

Historically, what Maxwell did was to take the work of Faraday, Gauss, Ampere and others, and pull them together in a series of equations that explain the relationships between $\vec{E}$ (electric) field and $\vec{H}$ (magnetic) field, and their auxiliary $\vec{D}$ (electric flux density) and $\vec{B}$ (magnetic flux density) fields. Oliver Heaviside gets credit for taking Maxwell's somewhat complex work and reducing it to the now-familiar (to engineers and physicists) four equations.

The first of Maxwell's equations shows that the divergence of the $\vec{D}$ field equals the volume charge density. This is Gauss' Law. (Note: equations are shown in their two common mathematical forms):

$$\nabla \cdot \overline{D} = \rho$$
$$\oint_S \overline{D} \cdot \overline{dS} = \oint_V \rho \, dv \qquad (1)$$

From [1], "When the $\vec{D}$ field lines exhibit a net departure (*divergence*) from any volume (even a microscopic one), there is a net positive electric charge inside. Vice versa, if the lines enter the volume, there is a net negative charge inside. Departing lines mean that a *positive charge* outside the volume will experience a force that repels is from the volume. This makes sense since *like charges repel*."

Equation two indicates that the $\vec{B}$ field has no divergence, in other words, they close in on themselves. This is why magnetic lines of force are always drawn as lines leaving a volume on path that eventually returns to that volume:

$$\nabla \cdot \overline{B} = 0$$
$$\oint_S \overline{B} \cdot \overline{dS} = 0 \qquad (2)$$

Also remember that $\vec{B}$ is proportional to $\vec{H}$, by the permeability of the of the medium, so $\vec{B} = \mu\vec{H}$.

The third equation introduces, the concept of curl, which is the amount circulation, or movement, of the vector field:

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$
$$\oint \overline{E} \cdot \overline{dl} = -\int_S \frac{\partial \overline{B}}{\partial t} \cdot \overline{dS} \qquad (3)$$

Since this is the incremental form of Faraday's law, this shows how a voltage is induced in a loop by a moving magnetic field, proportional to the rate of change of the field. The negative sign means that the polarity will oppose the change in the $\vec{B}$ field. This, of course, is the phenomenon that makes electric motors and transformers operate.

Finally, we have the fourth equation:

$$\nabla \times \overline{H} = \overline{J_C} + \frac{\partial \overline{D}}{\partial t}$$
$$\oint \overline{H} \cdot \overline{dl} = \oint_S \overline{J_C} \cdot \overline{dS} + \oint_S \frac{\partial \overline{D}}{\partial t} \cdot \overline{dS} \qquad (4)$$

which states (from [1]), "...that the curl of the $\vec{H}$ field is proportional to the enclosed current ... which has two components: (1) the conduction current density and (2) the time rate of change of the electric flux density ... (the displacement current density)."

Among the things this equation describes is antenna behavior, where current in the antenna conductor creates a time-varying $\vec{H}$ field in the surrounding space which, in turn, produces a time-varying $\vec{E}$ field perpendicular to the $\vec{H}$ field. Once one field is produced, the other is induced by the time rate of change of the other, which induces the other, and so on—a sequence that results in an electromagnetic wave propagating away from the source.

Reference

1. Joseph F. White, *High Frequency Techniques*, John Wiley & Sons, New Jersey, 2004. ISBN 0-471-45591-1.