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# Power Combiners, Impedance Transformers and Directional Couplers: Part IV

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This series of articles concludes with an examination of directional couplers. Readers are reminded that all four parts can be downloaded from our Web site.

## Coupled-Line Directional Couplers

The first directional couplers consisted of either a two-wire balanced line coupled to a second balanced line along a distance of quarter wavelength, or a pair of rods a quarter wavelength long between ground planes [53]. Although the propagation of waves on systems of parallel conductors was investigated many decades ago—in connection with the problem of crosstalk between open wire lines or cable pairs in order to eliminate the natural coupling rather than use it—the first exact design theory for TEM (transverse electromagnetic) transmission-line couplers was introduced by Oliver [74].

In terms of the even and odd electric-field modes describing a system of the coupled conductors, it can be stated that the coupling is backward with the coupled wave on the secondary line propagating in the direction opposite to the direction of the wave on the primary line, the directivity will be perfect with *VSWR* equal to unity if $Z_0^2 = Z_{0e}Z_{0o}$ at all cross sections along the directional coupler, and the midband voltage coupling coefficient $C$ of the directional coupler is defined as

$$C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \qquad (48)$$

A coupled-line directional coupler, the stripline single-section topology of which is shown in Figure 40(a), can be used for broadband power division or combining. Its electrical properties are described using a concept of two types of excitations for the coupled lines in TEM approximation. In this case, for the even mode, the currents flowing in the strip conductors are equal in amplitude and flow in the same direction. The electric field has even symmetry about the center line, and no current flows between the two strip conductors. For the odd mode, the currents flowing in the strip conductors are equal in amplitude, but flow in opposite directions. The electric field lines have an odd symmetry about the center line, and a voltage null exists between these two strip conductors. An arbitrary excitation of the coupled lines can always be treated as a superposition of appropriate amplitudes of even and odd modes. Therefore, the characteristic impedance for even excitation mode $Z_{0e}$ and the characteristic impedance for the odd excitation mode $Z_{0o}$ characterize the coupled lines. When the two coupled equal-length strip lines are used in a standard system with characteristic impedance of $Z_0$, $Z_0^2 = Z_{0e}Z_{0o}$ and

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}} \qquad (49)$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}} \qquad (50)$$

An analysis in terms of scattering $S$-parameters gives $S_{11} = S_{14} = 0$ for any electrical lengths of the coupled lines and the output port 4 is isolated from the matched input port 1. Changing the coupling between the lines and their widths can change the characteristic impedances $Z_{0e}$ and $Z_{0o}$. In this case,

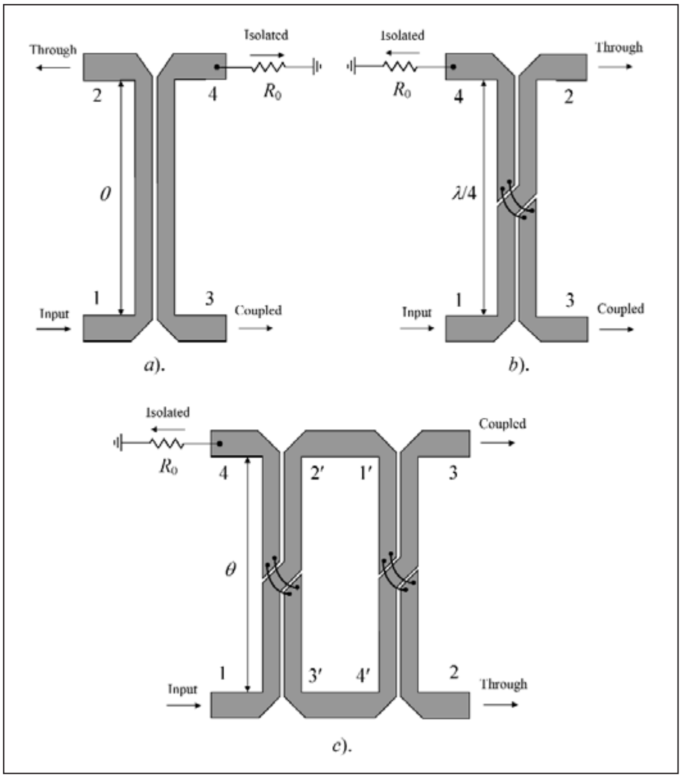


**Figure 40 · Coupled-line directional couplers.**

$$S_{12} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \quad (51)$$

$$S_{13} = \frac{jC\sin\theta}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \quad (52)$$

where $\theta$ is the electrical length of the coupled-line section.

The voltage-split ratio $K$ is defined as the ratio between voltages at port 2 and port 3 by

$$K = \left|\frac{S_{12}}{S_{13}}\right| = \frac{\sqrt{1-C^2}}{C\sin\theta} \quad (53)$$

where $K$ can be controlled by changing the coupling coefficient $C$ and electrical length $\theta$.

For a quarter-wavelength-long coupler when $\theta$ = 90°, Eqs. (51) and (52) reduce to

$$S_{12} = -j\sqrt{1-C^2} \quad (54)$$

$$S_{13} = C \quad (55)$$

from which it follows that equal voltage split between the output ports 2 and 3 can be provided with

$$C = 1/\sqrt{2}$$

If it is necessary to provide the output ports 2 and 3 at one side, it is best to use a construction of a microstrip directional coupler with crossed bondwires, as shown in Figure 40(b). The strip crossover for a stripline directional coupler can be easily achieved with the three-layer sandwich. A microstrip 3-dB directional coupler fabricated on alumina substrate for idealized zero strip thickness should have the calculated strip spacing of less than 10 µm. Such a narrow value easily explains the great interest in the construction of directional couplers with larger spacing. The effective solution is to use a tandem connection of the two identical directional couplers, which alleviates the physical problem of tight coupling, since two individual couplers need only 8.34-dB coupling to achieve a 3-dB coupler [75, 76]. The tandem coupler shown in Figure 40(c) has the electrical properties of the individual coupler when the output ports 1, 4 and 2, 3 are isolated in pairs, and the phase difference between the output ports 2 and 3 is of 90°.

From an analysis of the signal propagation from input port 1 to output ports 2 and 3 of the tandem coupler, when the signal from the input port 1 propagates to the output port 2 through the traces 1-2′-1′-2 and 1-3′-4′-2 while the signal flowing through the traces 1-2′-1′-3 and 1-3′-4′-3 is delivered to the output port 3, the ratio of the scattering parameters and $S_{12}^T$ and $S_{13}^T$ of a tandem coupler can be expressed through the corresponding scattering parameters $S_{12}$ and $S_{13}$ of the individual coupler as

$$\frac{S_{12}^T}{S_{13}^T} = \frac{S_{12}^2 + S_{13}^2}{2S_{12}S_{13}} = -j\frac{1-C^2\left(1+\sin^2\theta\right)}{2C\sqrt{1-C^2}\sin\theta} \quad (56)$$

As a result, the signal at the port 2 overtakes the signal at the port 3 by 90°. In this case, for a 3-dB tandem coupler with $\theta$ = 90°, the magnitude of Eq. (56) must be equal to unity. Consequently, the required voltage coupling coefficient is calculated as

$$C = 0.5\sqrt{2-\sqrt{2}} = 0.3827$$

or

$$C_{12} = C_{13} = 8.34 \text{ dB}$$

As an example, a tandem 8.34-dB directional coupler has the dimensions of $W/h$ = 0.77 and $S/h$ = 0.18 for alumina substrate with $\varepsilon_r$ = 9.6, where $W$ is the strip width, $S$ is the strip spacing and $h$ is the substrate thickness [56].

Another way to increase the coupling between the two edge-coupled microstrip lines is to use several parallel narrow microstrip lines interconnected with each other by the bondwires, as shown in Figure 41. For a Lange coupler shown in Figure 41(a), four coupled microstrip lines

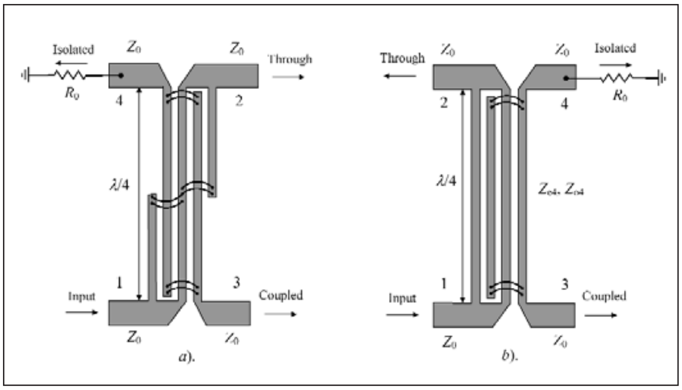


Figure 41 · Lange directional couplers: (a) original interdigital configuration, and (b) "unfolded" version.

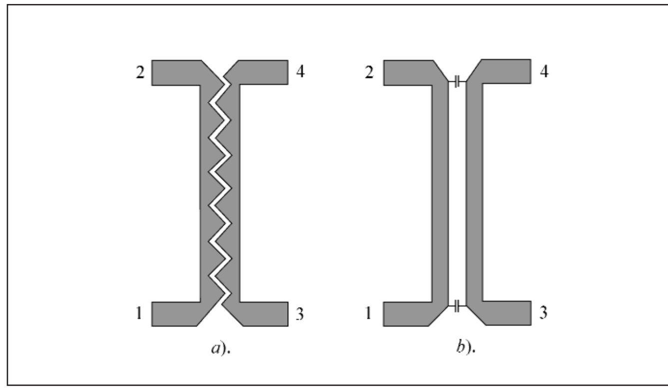


Figure 42 · Coupled-line directional couplers with compensation for directivity vs. frequency.

are used, achieving a 3-dB coupling over an octave or more bandwidth [77]. In this case, the signal flowing to the input port 1 is distributed between the output ports 2 and 3 with the phase difference of 90°. However, this structure is quite complicated for practical implementation when, for alumina substrate with $\varepsilon_r$ = 9.6, the dimensions of a 3-dB Lange coupler are $W/h$ = 0.107 and $S/h$ = 0.071, where $W$ is the width of each strip and $S$ is the spacing between adjacent strips.

Figure 41(b) shows the unfolded Lange coupler with four strips of equal length; it offers the same electrical performance but is easier for circuit modeling [78]. The even-mode characteristic impedance $Z_{e4}$ and odd-mode characteristic impedance $Z_{o4}$ of the Lange coupler with $Z_0^2 = Z_{e4}Z_{o4}$ in terms of the characteristic impedances of a two-conductor line (which is identical to any pair of adjacent lines in the coupler) can be obtained by

$$Z_{e4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0o} + Z_{0e}} Z_{0e} \quad (57)$$

$$Z_{o4} = \frac{Z_{0e} + Z_{0o}}{3Z_{0e} + Z_{0o}} Z_{0o} \quad (58)$$

where $Z_{0e}$ and $Z_{0o}$ are the even- and odd-mode characteristic impedances of the two-conductor pair [79].

The midband voltage coupling coefficient C is given by

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{04}} = \frac{3\left(Z_{0e}^2 - Z_{0o}^2\right)}{3\left(Z_{0e}^2 - Z_{0o}^2\right) + 2Z_{0e}Z_{0o}} \quad (59)$$

The even- and odd-mode characteristic impedances $Z_{0e}$ and $Z_{0o}$, as functions of the characteristic impedance $Z_0$ and coupling coefficient $C$, are determined by

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}}\,\frac{4C - 3 + \sqrt{9 - 8C^2}}{2C} \quad (60)$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}}\,\frac{4C + 3 - \sqrt{9 - 8C^2}}{2C} \quad (61)$$

For alumina substrate with $\varepsilon_r$ = 9.6, the dimensions of such a 3-dB unfolded Lange coupler are $W/h$ = 0.112 and $S/h$ = 0.08, where $W$ is the width of each strip and $S$ is the spacing between the strips.

The design theory for TEM transmission-line couplers is based on an assumption of the same phase velocities of the even and odd propagation mode. However, this is not the case for coupled microstrip lines, since they have unequal even- and odd-mode phase velocities. In this case, the odd mode has more fringing electric field in the air region, but with the even mode, the electric field is concentrated mostly in the substrate under the microstrip lines. As a result, the effective dielectric permittivity in the latter case is higher, thus indicating a smaller phase velocity for the even mode. Consequently, it is necessary to apply phase velocity compensation techniques to improve coupler directivity, which decreases with increasing frequency. Figure 42(a) shows a typical wiggly-line coupler (with sawtooth shape of coupled lines), which increases the physical lengths of the adjacent edges of the microstrip lines. This slows the odd-mode wave with little affect on the even-mode wave [80].

High directivity can also be achieved by using capacitive compensation. Figure 42(b) shows the capacitively compensated microstrip directional coupler where two identical lumped capacitors are connected between the coupled lines at their edges. Physically, these edge capacitors affect the odd mode by equivalent extension of the transmission-line electrical lengths, with almost no effect on even mode. For an ideal lossless operation condition at 12 GHz using standard alumina substrate, the compensated coupled-line microstrip directional coupler can improve directivity from 13.25 dB to infinity [81]. Capacitive compensation can also be performed by a gap

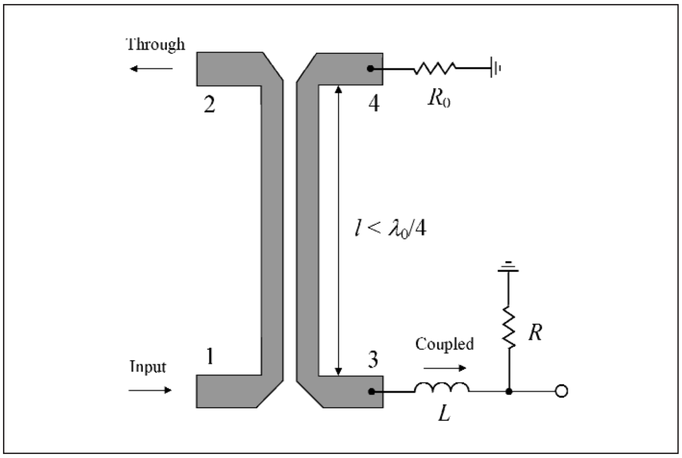


**Figure 43 · Coupled-line directional coupler with reduced-size and frequency compensation.**

coupling of the open-circuit stub formed in a stub-coupled line [82]. In this case, the coupler directivity can be improved by 23 dB in a frequency range from 1 to 2.5 GHz compared to the directivity of the conventional uncompensated microstrip coupler.

At radio frequencies and low microwaves, the conventional quarter-wavelength directional coupler has very large dimensions that limit their practical application especially in monolithic circuits. Figure 43 shows a reduce-size directional coupler consisting of the two coupled microstrip lines, the electrical lengths of which are much smaller than quarter wavelength. The main problem of the coupler at frequencies where the electrical length of its coupled lines is smaller than quarter wavelength, is that the degree of coupling linearly varies with frequency. To compensate this frequency behavior, the output port 3 can be connected to a series inductor $L$ followed by a shunt resistor $R$ [36, 83]. The inductance value depends on the coupling value and flatness, and midband frequency, while the resistance value depends on the impedance of the secondary line and inductance value. Such a microstrip reduced-size directional coupler with $L = 180$ nH and $R = 62\ \Omega$ can provide the coupling of about 30 dB with flatness of ±0.1 dB, directivity greater than 20 dB, insertion loss less than 0.25 dB, and *VSWR* less than 1.15 in a frequency bandwidth of 60% around 200 MHz. Tuning of the center bandwidth frequency and coupling can be simply realized by varying the inductance value.

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*Editor’s note—This is the final article in a four-part series on combiners, transformers and couplers. The previous three articles, published in the past three issues, are available for dowloading from the Archives section of www.highfrequencyelectronics.com.*