Transmission Line Transformers: Theory, Design and Applications — *Part 2*

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This two-part article concludes with a discussion of magnetic materials and practical realization of these transformers

Magnetic Materials

The introduction of magnetic materials such as powdered iron or ferrite improves the low-frequency bandwidth limit by increasing the magnetically induced inductance of the conductors, which can be approximated by:

$$L'_{ac} = L_{ac}\sqrt{\mu_r} \qquad (29)$$

At lower frequencies the response is dominated by the magnetising inductance for all windings [3, 6]. The use of magnetic materials to increase the impedance seen by common-mode currents and to improve the coupling of odd-mode currents in TLTs can be traced back to Guanella [29].

Also, as a general rule we look at the magnetic material as increasing the length of the transmission line by approximately:

$$l' = l\sqrt{\mu_r} \qquad (30)$$

where l' is the apparent length of the transmission line and l is the actual physical length. This approximation is more appropriate for TLTs made with twisted wire or parallel wires as they are more directly influenced by the magnetic material due to stray coupling.

When using coaxial cable transmission line in a TLT, the skin effect of the inside surface of the outer conductor causes the current on the outer conductor to be concentrated on the inside surface. The magnetic fields generated by the equal and opposite currents on the inner conductor and the inner surface of the outer conductor cancel outside of the outer conductor, leaving no net magnetic field outside the outer conductor and therefore no magnetic flux in the core. Thus, when the coaxial cable is functioning as a TEM mode transmission line the magnetic core will cause no additional losses beyond those of the cable itself [17, 22].

Additionally, the TEM mode works as long as the currents on the outside surface of the outer conductor of the coaxial cable are negligible [6], and the considerable magnetic losses of the magnetic material dissipates these outside surface currents, thereby improving the high frequency bandwidth limit. Several transmission lines of a transformer may be

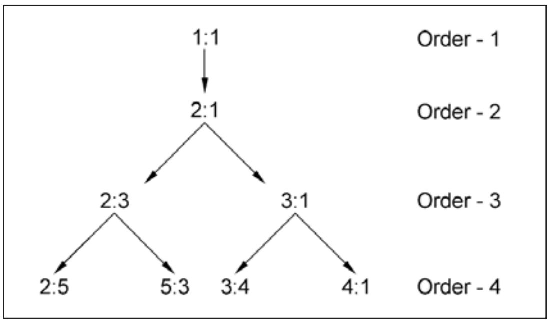

Figure 8 · Voltage transformation ratios of the first four orders of TLTs (from Rotholz (7)).

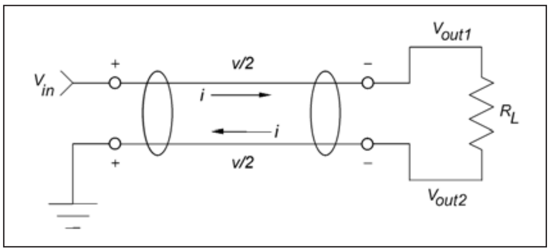

Figure 9 · Order-1 TLT used as a 1:1 choke balun.

wound on a single magnetic core, as was demonstrated by Ruthroff [7, 30], provided that the voltages and currents of each transmission line in the transformer are identical.

In practice, the magnetic material is selected so that the ferroresonance frequency of the magnetic material is above the low frequency bandwidth limit of the transmission line by itself.

TLT Order and Synthesis

The number of transmission lines which comprise a TLT is the order of the TLT. An order-m TLT is a two-terminal pair device which consists of m connected lines. A TLT of order (m + 1) is obtained by connecting an additional transmission line to the terminals of an order-m TLT, in parallel at one end and in series at the other [7]. The voltage transformation ratios of the first 4 orders of TLTs are shown in Figure 8 [7].

A TLT of order-1 is a single transmission line [7], and an application of order-1 TLTs is shown in Figure 9 as a device commonly referred to as a choke balun, where the floating load Z_o receives the equal and opposite currents from the output terminals of the TLT, and which was originally patented by Gerth [31, 32] and later by Guanella [33].

Earlier, we recognized that the second terms of equations (15) and (16) denoted a voltage along the length of the transmission line and that this voltage is equal in magnitude and phase for both conductors. This was later illustrated in the low frequency models of Figures 6 and 7, and the convention of the voltage along the length of the lines will be used from this point forward as it is more convenient in the voltage and current mapping of the TLTs. Since the voltages along the length of both sides of the TLT of Figure 9 must be equal, the voltage along the length of the TLT is half the input voltage, therefore causing the output terminal voltages to be $+v/2$ and $-v/2$, and the input impedance and output voltages of Figure 9 are described as:

$$Z_{\text{in}} = Z_0\left(\frac{R_L \cosh\gamma I + Z_0 \sinh\gamma I}{Z_0 \cosh\gamma I + R_L \sinh\gamma I}\right) \tag{31}$$

$$V_{\text{out1}} = \frac{R_L}{2}\frac{V_{\text{in}}}{R_L \cosh\gamma I + Z_0 \sinh\gamma I} \tag{32}$$

$$V_{\text{out2}} = -\frac{R_L}{2}\frac{V_{\text{in}}}{R_L \cosh\gamma I + Z_0 \sinh\gamma I} \tag{33}$$

which does not include the loss(es) and magnetization inductance(s) of the low frequency models of Figures 6 and 7.

This convenient form of balun will obviously work equally well with symmetrical (balanced) loads, and will also work with asymmetrical and unbalanced loads, such as is shown in Figure 10 where the 1:1 choke balun of Figure 9 is used as a 1:1 phase inverter [22, 24, 34], the input impedance and output voltage of which is:

$$Z_{\text{in}} = Z_0\left(\frac{R_L \cosh\gamma I + Z_0 \sinh\gamma I}{Z_0 \cosh\gamma I + R_L \sinh\gamma I}\right) \tag{34}$$

$$V_{\text{out2}} = \frac{-R_L V_{\text{in}}}{R_L \cosh\gamma I + Z_0 \sinh\gamma I} \tag{35}$$

which also does not include the loss(es) and magnetization inductance(s) of the low frequency models of Figures 6 and 7.

An order-1 TLT is also shown in Figure 11 as a 1:1 current balun, which has output currents that are equal in magnitude and opposite in phase regardless of the potentials at the output terminals with regard to the ground connection on the unbalanced (input) side [30]. This is a more interesting TLT to describe mathematically, as not only do the voltages at the ends of the transmission line need to be equal, but since there is no ground connection on the output side, the currents at both ends of the transmission line also need to be equal. Consequently, the input impedance and output voltages are described as:

$$Z_{\text{in}} = \frac{Z_0 R_L \cosh\gamma I + \left(n(1-n)\right) R_L^{\,2} \sinh\gamma I}{\left(Z_0 \cosh\gamma I + n\, R_L \sinh\gamma I\right)} \tag{36}$$

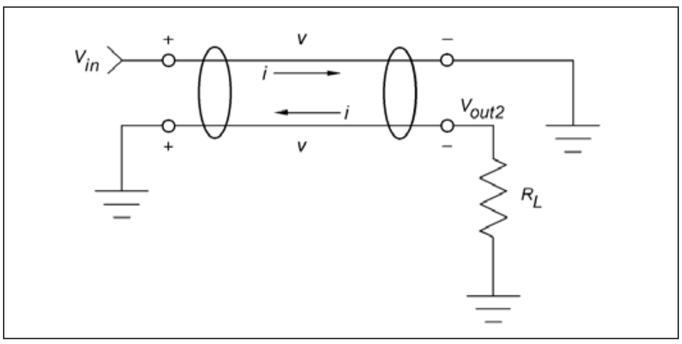

Figure 10 · Order-1 TLT used as a 1:1 phase inverter.

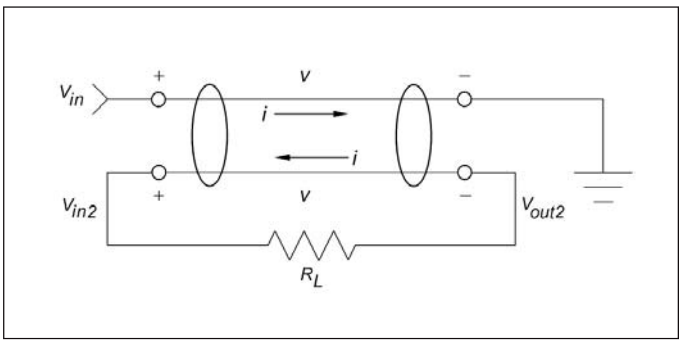

Figure 11 · Order-1 TLT used as a 1:1 current balun.

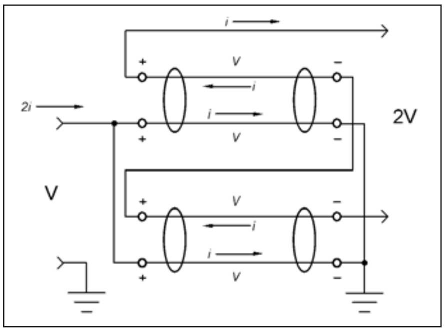

Figure 12 · Order-2 TLT used as a 2:1 current (1:4 impedance) balun.

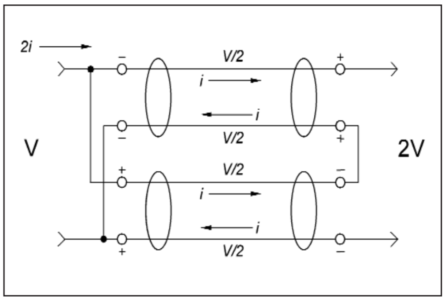

Figure 14 · The Guanella 1:4 impedance ratio TLT, which uses two lines.

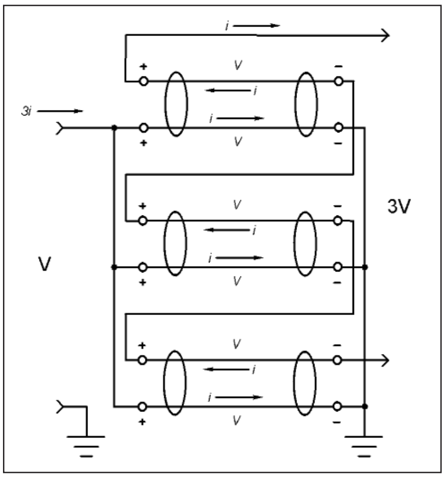

Figure 13 · Order-3 TLT used as a 3:1 current (1:9 impedance) balun.

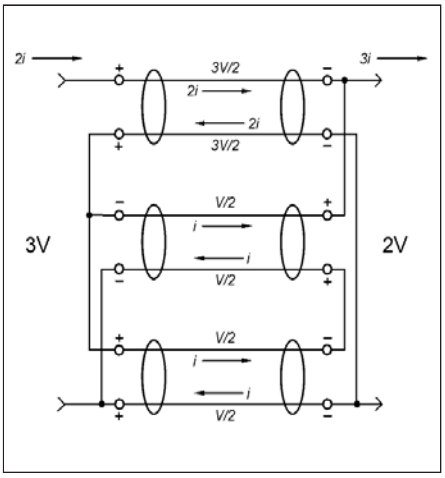

Figure 15 · Order-3 TLT having a 3:2 voltage (2.25:1 impedance) ratio.

$$V_{\text{in2}} = \frac{V_{\text{in}}(1-n)R_L Z_0}{Z_0 R_L \cosh\gamma\, l + \big(n(1-n)\big)R_L^{\,2}\sinh\gamma\, l} \qquad (37)$$

$$V_{\text{out2}} = \frac{-V_{\text{in}} n\, R_L Z_0}{Z_0 R_L \cosh\gamma\, l + \big(n(1-n)\big)R_L^{\,2}\sinh\gamma\, l} \qquad (38)$$

where

$$n = \frac{Z_0}{R_L}\left|\frac{\cosh\gamma\, l - 1}{\sinh\gamma\, l}\right| = \frac{Z_0}{R_L}\tanh\frac{\gamma\, l}{2} \qquad (39)$$

which again does not include the loss(es) and magnetization inductance(s) of the low frequency models of Figures 6 and 7. The order-1 TLTs of Figures 9 and 11 are two distinct realizations, which we will refer to as first and second canonical form, respectively. We will later see that these distinct forms can result in alternate realisations of higher order TLTs.

TLTs of r:1 voltage ratio are the simplest configuration when they consist of r transmission lines, all of them connected in series at one end and in parallel at the other [7]. Figures 12 and 13 show the connections for TLTs having voltage (and current) ratios of 1:2 (1:4 impedance) and 1:3 (1:9 impedance), respectively. In Figure 12 it is obvious from the voltages and currents that the output voltage is twice the input voltage, and the input current is twice the output current, so the impedance ratio is 1:4 (1: r^2). A similar analysis can be followed for Figure 13. Since the voltages and currents for all sections of the 1:r voltage ratio are identical, this class of order-r TLT can be constructed on a single core.

The r:1 voltage ratio TLTs of Figures 12 and 13 are of the second canonical form as they are realized by way of adding successive second canonical form order-1 TLTs of Figure 11. TLTs of r:1 voltage ratio may also be realised by way of adding successive first canonical for order-1 TLTs of Figure 9, but these will more often than not require multiple cores and are therefore less attractive.

The synthesis procedure for an arbitrary integer voltage ratio is fairly simple. An H:L ($H > L$) voltage ratio TLT is decomposed into an (H-L):L ratio TLT and a transmission line which is connected in series with the (H-L) side and in parallel with the L side. The procedure is repeated until a 1:1 order-1 TLT is reached [7].

The Guanella 1:4 Impedance Ratio TLT

Figure 14 illustrates the connections for another form of order-2 TLT having a 1:2 voltage transformation ratio (1:4 impedance ratio) commonly known as the Guanella 4:1 impedance ratio transformer [1, 2, 5, 26], where an additional transmission line has been added to the order-1 1:1 choke balun of Figure 8. This 2:1 voltage ratio TLT is therefore of the first canonical form.

Notice here that the added transmission line is connected in parallel on the left and in series on the right. This is a very popular device, and when used with a floating load such as an antenna, the voltages and currents for the two transmission lines are identical, as shown in Figure 14, which will allow it to be constructed on a single core. In general, the Guanella 1:4 impedance transformer requires that each of the transformer sections be constructed on separate cores as the voltages are dissimilar even though the currents are the same.

Higher Order Realizations

Figure 15 illustrates the connections for an order-3 TLT having a voltage ratio of 3:2 (impedance ratio of 2.25:1), where an additional transmission line has been added to the order-2 Guanella 1:2 voltage ratio TLT of Figure 14. Notice here that the added transmission line is connected in parallel on the right side and in series on the left.

Figure 16 illustrates the connections for an order-4 TLT having a voltage ratio of 3:5 (impedance ratio of 1:2.78), where an additional transmission line has been added to the order-3 3:2 voltage ratio TLT of Figure 15. Notice here that the added transmission line is connected in parallel on the left side and in series on the right.

Additional forms of order-2, order-3, and order-4 TLTs are possible, the ones shown here being used as examples. Many rigorous synthesis procedures have been published, such as those by McClure [27, 28], Myer [36], and Gluszczak [37], as well as a synthesis procedure for designing TLTs having fractional transformation ratios [38].

Closing Remarks

The design and application of transmission line transformers using multiple sections of transmission line having identical lengths and characteristics is an established element of RF circuit design and has been part of the technology for over 60 years, beginning as early as the impedance matching transformer published by Guanella in 1944 [1, 2]. TLT realizations using magnetic materials to extend the low and high bandwidth limits can have frequency bandwidths of four decades or more [11, 29]. Transmission line transformers encompass the aspects of wide bandwidth and low losses which are highly desireable features for the design of high-power RF components. The technology and synthesis procedures are readily available and easily understood even by those having entry level experience in the profession of RF circuit design.

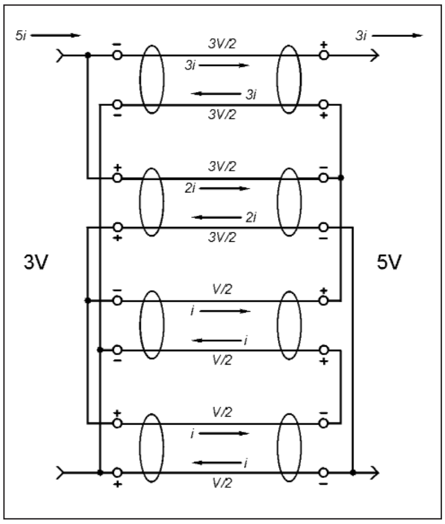

Figure 16 · Order-4 TLT having a 3:5 (1:2.78 impedance) ratio.

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