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SPICE Models with Frequency Dependent Conductor and Dielectric Losses

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High frequency analysis using SPICE requires models that accurately represent frequency-dependent behaviors such as resistive and dielectric losses

Over the past two decades, various methods have been used to adapt frequency-domain behaviors to the time-domain environment of circuit modeling with SPICE. This article will discuss the creation of SPICE Models that account for frequency dependent conductor and dielectric losses using the Boundary Element Method (BEM) to solve the field equations, along with a new universal frequency dependent lossy SPICE model generator which enables accurate simulation in both time and frequency domain within any SPICE platform.

SPICE Loss Models

Network synthesis techniques are used to develop RL and RC Cauer network formulations for both the skin effect conductor losses and the "tan δ" dielectric losses which are positive real impedance or admittance functions that have real parts that behave monotonically as functions of frequency. The loss models are developed over a frequency range bounded by the lower corner frequency characteristic to the modeled loss and an integer number of decades to the maximum frequency specified by the user.

The formulations of the real parts of the impedance or admittance functions for a Cauer network and the corresponding derivative functions are presented in Appendix I. The impedance function is for a series chain of parallel RL elements and the admittance function is for a parallel combination of series RC elements. The number of elements in each representation is equal to the number of decades spanned by the model. The resulting loss sections are partitioned out over the length of the SPICE sub-circuit, which can be either a "ladder" or "nodal" model and the resultant lossy sub-circuit model can be run on any SPICE solver platform!

Skin Effect Conductor Losses

The skin effect conductor losses vary with the square root of frequency. The residues K_i, as defined in Appendix I, are evaluated by equating the real part of the Cauer network impedance function and derivatives to the real part of the loss impedance function, as produced by the solver, and its derivatives for poles, σ_i, at each decade between the lower corner frequency of the loss function and the maximum frequency, f_m, selected by the user. The lower corner frequency, $f_{0,c}$, is related to the geometry of the conductor under consideration by:

$$f_{0,c} = \frac{R_0 C^2}{\pi \mu_0 A}$$

where R_0 is the conductor DC resistance per unit length, C is the conductor perimeter, and A is the cross sectional area of the conductor.

It is also related to the conductor resistance per unit length, R_m, at the user defined maximum frequency, f_m, by:

$$f_{0,c} = \left(\frac{R_0}{R_m}\right)^2 f_m$$

where R_m is calculated by the field solver. For frequencies less than $f_{0,c}$ the losses are constant at the DC value.

Once the residues are known, the corresponding parallel RL elements for each decade or σ_i are calculated—yielding a series chain of parallel RL elements which matches the conductor loss impedance function over the frequency range from DC to f_m. These series network elements are partitioned out over the length of the ladder or nodal SPICE sub-circuit model.

Tanδ Dielectric Losses

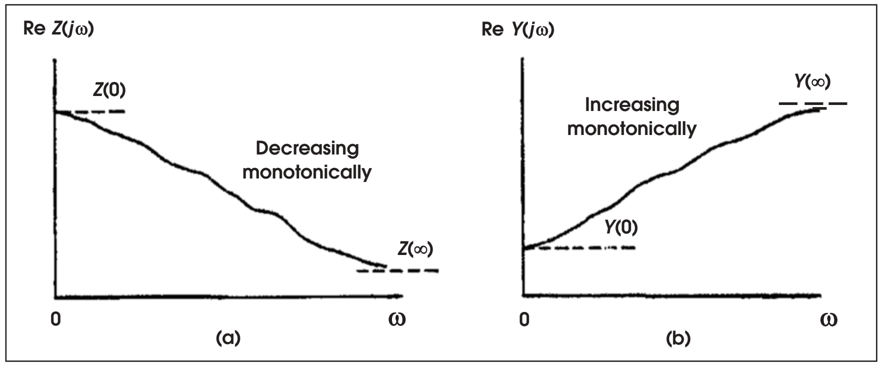
The tanδ dielectric losses, for a constant loss tangent, vary linearly with frequency. The residues K_i', as defined in Appendix I, are evaluated by equating the real part of the Cauer network admittance function and derivatives to the real part of the dielectric loss admittance function, as produced by the solver, and its derivatives for poles, σ_i, at each decade between the lower corner frequency of the loss admittance function and the maximum frequency, f_m, selected by the user. The lower corner frequency for the dielectric loss function is related to the transmission characteristics of the modeled cross-section by

$$f_{0,d} = \frac{1}{2\pi L\tau}$$

where L is the transmission line cross-section length, and τ is the section time delay per unit length as calculated by the solver for the conductor under consideration. For frequencies less than $f_{0,d}$ the modeled losses are zero. DC conductance losses can be included by the user outside of this loss model.

Again, once the residues are known, the corresponding series RC elements for each decade or σ_i are calculated—yielding a parallel combination of series RC elements which matches the dielectric loss admittance function from DC to f_m. As with the conductor loss elements, these

Appendix I



The variation of Re *Z(jω)* and Re *Y(jω)* with ω for RC networks. For RL networks, exchange *Y* and *Z*.

"For RC networks, we now know that $Z(0) > Z(\infty)$ and $Y(\infty) > Y(0)$; for RL networks, $Z(\infty) > Z(0)$ and $Y(0) > Y(\infty)$. These relations are basic in the synthesis method which results in ladder network structures which we will study next. Since both the impedance and the admittance functions for RC networks have real, positive values and zero and infinity, it follows that Re $Z(j0) >$ Re $Z(j\infty)$ and that Re $Y(j\infty) >$ Re $Y(j0)$ since $Z(\sigma)$ with $\sigma = 0$ is the same as $Z(j\omega)$ with $\omega = 0$, and $Z(\sigma)$ with $\sigma = \infty$ is the same as $Z(j\omega)$ with $\omega = \infty$, i.e. both zero and infinity are each one point in the s plane. Now, to remove a constant from $Z(s)$ and have the resulting impedance remain positive real, it is necessary that the constant be less than or equal to the minimum value that Re $Z(j\omega)$ attains for all positive ω. We can find the values for Re $Z(j0) = Z(0)$ and Re $Z(j\infty) = Z(\infty)$. But what about intermediate values of Re $Z(j\omega)$ between 0 and ∞ ?" From *Introduction to Modern Network Synthesis* by Van Valkenberg, Ch. 6 [1].

The Cauer form of Re $Z(j\omega)$ and Re $Y(j\omega)$, using primes to distinguish residues for the two cases, are:

$$\operatorname{Re} Z(j\omega) = \sum_{i=2}^{n} \frac{\sigma_i K_i}{\omega^2 + {\sigma_i}^2} + K_\infty$$

and

$$\operatorname{Re} Y(j\omega) = \sum_{i=1}^{m} \frac{K_i' \omega^2}{\omega^2 + {\sigma_i}^2} + K_0$$

The derivatives of these functions are:

$$\frac{d}{d\omega} \operatorname{Re} Z(j\omega) = \sum_{i=2}^{n} \frac{-2K_i \sigma_i \omega}{\left(\omega^2 + {\sigma_i}^2\right)^2}$$

$$\frac{d}{d\omega} \operatorname{Re} Y(j\omega) = \sum_{i=1}^{m} \frac{2K_i' {\sigma_i}^2 \omega}{\left(\omega^2 + {\sigma_i}^2\right)^2}$$

parallel network elements are partitioned out over the length of the SPICE sub-circuit model.

Conclusion

The resulting lossy SPICE sub-circuit model derived by network synthesis techniques provides an accurate and economical conductor and dielectric loss modeling capability in both the time and frequency domains, including correct time domain risetime degradation, and loss induced pulse distortion and dispersion. Examples of the SPICE frequency domain simulations of S21 for a lossy cable are shown in Appendix II. These lossy SPICE sub-circuits can be run on any SPICE solver platform.

Contact Information

Implementations of the lossy model are available in the Interactive Products Corporation's tool suite that includes IFSPro and IFS Connect. For more information about this subject, or to obtain a full trial version of the application, please visit the Interactive Products Corp. web site at www.interactive-products.com, or send an e-mail to sales@interactive-products.com. The company may also be reached by telephone at 919-280-8846.

References

1. M. E. Van Valkenburg, *Introduction to Modern Network Synthesis*, John Wiley & Sons, New York (1960), pp. 150-151.
2. E. P. Sayre, M. A. Baxter and T. Savarino, "Development of a New Transmission Line Skin Effect Model for SPICE Evaluations," Digital Communications Systems Design Conference, '97 Design SuperCom.
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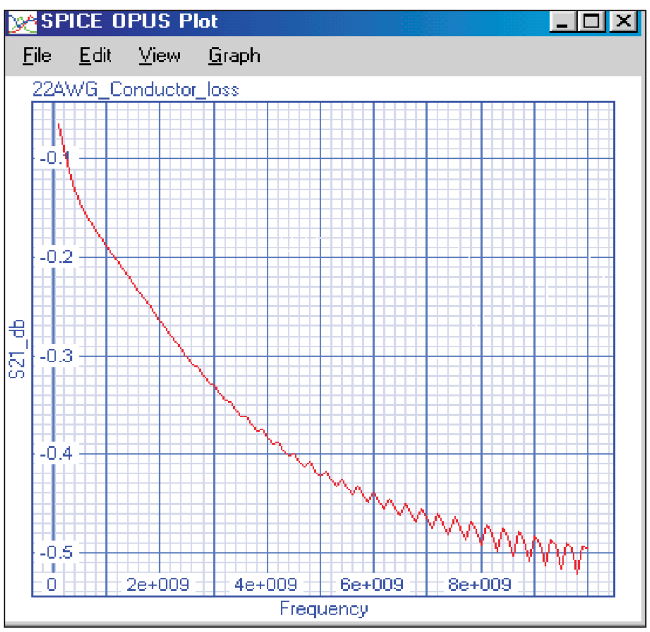
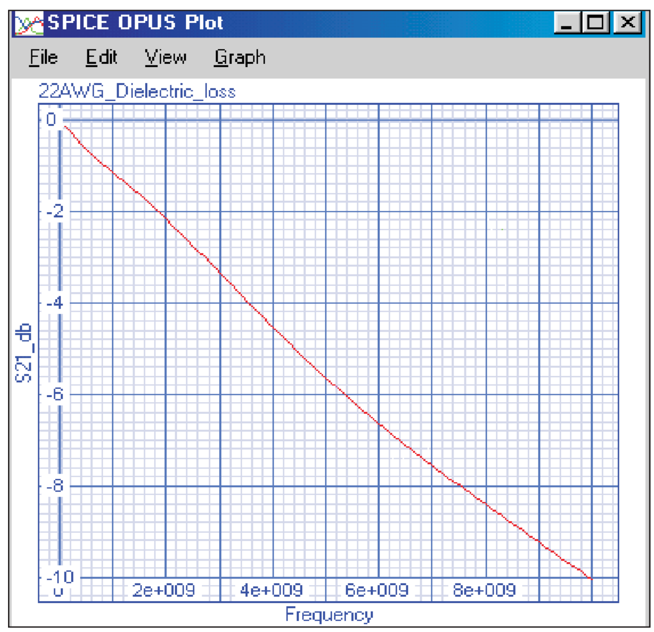
Appendix II

SPICE Frequency Domain Losses from Loss Models



Conductor losses only, for a 50 ohm 22 AWG coax 0.5 meters long.



Conductor and dielectric losses for a 50 ohm 22 AWG coax 0.5 meters long having a dielectric loss tangent of 0.01.