Basic Electromagnetics Trued and Cleared for All to See

By W. Scott Bennett, Ph.D.

> If current elements are truly viewed using concrete mathematics, basic electromagnetics are much easier to understand.

Note

HFE occasionally publishes material such as this article by Dr. Bennett which can be characterized as "exploratory" or "speculative." Although Dr. Bennett is challenging traditionally accepted theory, his intent is to find improved insight into electromagnetic understanding.

Introduction

The electric and magnetic fields of a current element are characterized by viewing it as a differential segment of a current—a nanocurrent. It is not viewed as it was by Heinrich Hertz, as a differential dipole antenna. Then, using a concrete description of each field of the nanocurrent, three of James Maxwell's abstract field equations are seen to be incorrect. Thus, if current elements are truly viewed using concrete mathematics, basic electromagnetics are made much easier to understand, and it is also seen that there is a need for correction.

A Nanocurrent and Its Fiel ds

A nanocurrent, $i(t)d\ell$ is any chosen differential segment of an electric current—a current element. However, Heinrich Hertz viewed $i(t)d\ell$ as an isolated differential dipole to find its electric and magnetic fields. And, any full-sized current has been viewed as superpositioned Hertzian dipoles to find its fields for a century, or more. However, nanoscale electronics have created a critical need for current elements to be truly viewed, and to establish near-field accuracy in their field descriptions.

The fields of a nanocurrent are caused by the electric charge movement in each differential cross section dA of its differential length $d\ell$ and by the net charge that forms between the ends of $d\ell$. Charge movement between the ends of $d\ell$ is

$$i(t)d\ell = \frac{dq(t)}{dt}d\ell = dq(t)\frac{d\ell}{dt} \qquad \left(\text{Coulombs x } \frac{\text{meters}}{\text{second}}\right)$$

And, if $d\ell$ Is centered at (0,0,0) in an otherwise empty medium, and $i(t)d\ell$ has the direction $\theta=0$ then at any given observation point (r, θ, ϕ), the charge movement $i(t)d\ell$ C×m/s) causes the ϕ-directed magnetic field

$$H_\phi(\tau) = \frac{\sin\theta}{4\pi r^2} i(\tau)d\ell \qquad \text{(Amperes/meter)}$$

where

$i(t)$ (Amps) is both the *midpoint current* and the *average current* in $d\ell$;
$4\pi r^2$ (m^2) is an *imaginary spherical surface area* with $i(t)d\ell$ at its *center;*
$\tau = t - t_p$ was the *cause time* of $H_\phi(\tau)$ at $(0,0,0)$, and its *emission time*;
$t_p = r/v_p$ (sec) is the *field propagation time* from $(0,0,0)$ to (r,θ,ϕ); and
v_p (m/s) is the *field propagation velocity in the medium containing* $i(t)d\ell$.

A time-varying charge flow, or current, of $i(t)$ (C/s) causes a time-varying charge movement of $i(t)d\ell$ (C×m/s/s) that causes the additional magnetic field

$$\underline{H}_\phi(\tau) = t_p H'_\phi(\tau) = \frac{\sin\theta}{4\pi r^2} t_p i'(\tau)d\ell \qquad \text{(A/m)}$$

together with the electric field

$$\underline{E}_\theta(\tau) = Z_m \underline{H}_\phi(\tau) = Z_m \frac{\sin\theta}{4\pi r^2} t_p i'(\tau)d\ell \qquad \text{(V/m)}$$

Where Z_m (Ohms) is called the *characteristic impedance,* or *intrinsic impedance,* of the medium containing $i(t)d\ell$ and its fields.

The difference in end-currents, $di(t)/d\ell \neq 0$ causes the net charge

$$Q(t) = d\ell \int \frac{di(t)}{d\ell} dt = \frac{d\ell}{v_{pc}} \int di(t) = \frac{i(t)d\ell}{v_{pc}} \qquad \text{(C)}$$

to form in $d\ell$ [1,2], where v_{pc} (m/s) is the field propagation velocity in the conductor of $i(t)$. And, the net charge $Q(t) = i(t)d\ell/v_{pc}$ (C) causes the electric field

$$E_r(\tau) = \frac{Z_m v_p}{4\pi r^2} Q(\tau) = \frac{Z_m k}{4\pi r^2} i(\tau)d\ell \qquad \text{(V/m)}$$

where

$k = v_p / v_{pc}$. And, as it is with charge movement, a time-varying net charge of

$i'(t)d\ell / v_{pc}$ (C/s) causes another electric field

$$\underline{E}_r(\tau) = \frac{Z_m k}{4\pi r^2} t_p i'(\tau) d\ell \quad \text{(V/m)}$$

So, in an otherwise empty medium, the fields of a nanocurrent $i(t)d\ell$ will be

$$\boldsymbol{H}_\phi(\tau) = H_\phi(\tau) + \underline{H}_\phi(\tau) = \frac{\sin\theta}{4\pi r^2}[i(\tau) + t_p i'(\tau)]d\ell \quad \text{(A/m)}$$

$$\boldsymbol{E}_r(\tau) = E_r(\tau) + \underline{E}_r(\tau) = \frac{Z_m k}{4\pi r^2}[i(\tau) + t_p i'(\tau)]d\ell \quad \text{(V/m)}$$

and

$$\boldsymbol{E}_\theta(\tau) = \underline{E}_\theta(\tau) = Z_m \underline{H}_\phi(\tau) = Z_m \frac{\sin\theta}{4\pi r^2}[t_p i'(\tau)]d\ell \quad \text{(V/m)}$$

Also, in practice, few mediums are entirely empty, and changes may occur in Z_m and v_p with changes in (r, θ, ϕ). Therefore, any such changes should be watched for, and accounted for, as necessary.

A Hertzian Dipole and Its Fields

The current element $i(t)d\ell$ was viewed as a differential dipole by Heinrich Hertz, because its fields would then satisfy James Maxwell's field equations. Hertz also assumed the end-to-end current to be $i(t)$ in $d\ell$ so that it would have a net charge of

$q(t) = \int(i(t) - 0)dt$ (C) on one end, and $-q(t) = \int(0 - i(t))dt$ (C) on the other end.

Also, a current element so-viewed would have a moving charge of $i(t)d\ell$ (C×m/s), the same as a nanocurrent.

Therefore, because its fields are each viewed as leaving the midpoint of $d\ell$, using its nanocurrent fields

$\boldsymbol{H}_\phi(\tau)$, $\boldsymbol{E}_\theta(\tau)$, and $\boldsymbol{E}_r(\tau)$, a Hertzian dipole's fields would be

$$\boldsymbol{H}_{D\phi}(\tau) = \boldsymbol{H}_\phi(\tau) = \frac{\sin\theta}{4\pi r^2}[i(\tau) + t_p i'(\tau)]d\ell \quad \text{(A/m)}$$

$$\boldsymbol{E}_{Dr}(\tau) = \frac{2\cos\theta d\ell}{r}\boldsymbol{E}_r(\tau) = \frac{2\cos\theta d\ell}{r\, d\ell}\frac{Z_m k}{4\pi r^2}[i(\tau) + t_p i'(\tau)]d\ell \quad \text{(V/m)}$$

and

$$\begin{aligned}\boldsymbol{E}_{D\theta}(\tau) &= \frac{\sin\theta d\ell}{r}\boldsymbol{E}_r(\tau) + \boldsymbol{E}_\theta(\tau) \\ &= \frac{\sin\theta d\ell}{r}\frac{Z_m k}{4\pi r^2}[i(\tau) + t_p i'(\tau)]d\ell + Z_m \frac{\sin\theta}{4\pi r^2}[t_p i'(\tau)]d\ell \quad \text{(V/m)}\end{aligned}$$

These are the fields of a differential dipole viewed as two nanocurrents with $i(t)$ at its midpoint where its nanocurrents join, and zero current on their open ends. And, as shown in [2], these equal the textbook equations given for Hertzian dipole fields for many decades—further implying that it should be viewed as two nanocurrents.

Hertzian dipoles have long been used as current elements, because when viewed in superposition, they form true points of net charge and moving charge. However, each point of net charge is separated from the points of moving charge that cause it by

$d\ell/2$, but each dipole field is viewed as leaving its midpoint. Therefore, for $r >> 0$

superposed Hertzian dipoles do give accurate field descriptions, but for $r \to 0$ those field descriptions are not accurate. So, to engineer electromagnetics in nanoscale electronics, current elements cannot be viewed as Hertzian dipoles.

Maxwell's Equations

Two of James Maxwell's four abstract equations for the fields of any point source with the magnetic field

$\boldsymbol{H}(\tau)$, and the electric field $\boldsymbol{E}(\tau)$ are

$$\nabla \times \boldsymbol{H}(\tau) = \frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}(\tau)}{\partial t} \quad \text{and} \quad \nabla \times \boldsymbol{E}(\tau) = -\frac{Z_m}{v_p}\frac{\partial \boldsymbol{H}(\tau)}{\partial t}$$

and the other two of Maxwell's equations are

$$\nabla \cdot \boldsymbol{H}(\tau) = 0 \quad \text{and} \quad \nabla \cdot \boldsymbol{E}(\tau) = 0$$

And, for any $\boldsymbol{A} = (A_r, A_\theta, A_\phi)$, $\nabla \times \boldsymbol{A} = (\nabla_r \times \boldsymbol{A}, \nabla_\theta \times \boldsymbol{A}, \nabla_\phi \times \boldsymbol{A})$, where

$$\nabla_r \times \boldsymbol{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]$$

$$\nabla_\theta \times \boldsymbol{A} = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right]$$

and

$$\nabla_\phi \times \boldsymbol{A} = \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]$$

Thus, for the fields $\boldsymbol{H}(\tau) = (0, 0, \boldsymbol{H}_\phi(\tau))$ and $\boldsymbol{E}(\tau) = (\boldsymbol{E}_r(\tau), \boldsymbol{E}_\theta(\tau), 0)$ of any chosen nanocurrent $i(t)d\ell$,

$$\nabla \times \boldsymbol{H}(\tau) = \left(\frac{1}{r\sin\theta}\frac{\partial(\sin\theta \boldsymbol{H}_\phi(\tau))}{\partial\theta}, -\frac{1}{r}\frac{\partial(r\boldsymbol{H}_\phi(\tau))}{\partial r}, 0\right)$$

and

$$\frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}(\tau)}{\partial t} = \left(\frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}_r(\tau)}{\partial t}, \frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}_\theta(\tau)}{\partial t}, 0\right)$$

However, because the nanocurrent field $\boldsymbol{E}_r(\tau)$ is independent of θ,

$$\nabla_r \times \boldsymbol{H}(\tau) = \frac{1}{r\sin\theta}\frac{\partial(\sin\theta \boldsymbol{H}_\phi(\tau))}{\partial\theta} \neq \frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}_r(\tau)}{\partial t}$$

So, for any $i(t)d\ell$

$$\underline{\nabla \times \boldsymbol{H}(\tau) \neq \frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}(\tau)}{\partial t}}$$

Also, because $\partial \boldsymbol{E}_r(\tau)/\partial\theta = 0$,

$$\begin{aligned}\nabla_\phi \times \boldsymbol{E}(\tau) &= \frac{1}{r}\frac{\partial(r\boldsymbol{E}_\theta(\tau))}{\partial r} = \frac{1}{r}\frac{\partial(r\underline{E}_\theta(\tau))}{\partial r} \\ &= \frac{Z_m}{r}\frac{\partial(r\underline{H}_\phi(\tau))}{\partial r} = Z_m\left(\frac{\underline{H}_\phi(\tau)}{r} + \frac{\partial \underline{H}_\phi(\tau)}{\partial r}\right)\end{aligned}$$

And, noting that $t_p = r/v_p$, $\tau = t - r/v_p$, and $\partial i'(\tau)/\partial r = -\frac{1}{v_p}\partial i'(\tau)/\partial t$, it follows that

$$\begin{aligned}
\frac{\partial \underline{H}_\phi(\tau)}{\partial r} &= \frac{\partial}{\partial r}\left(\frac{\sin\theta}{4\pi r^2} t_p i'(\tau) d\ell\right) = \frac{\sin\theta d\ell}{4\pi v_p}\frac{\partial}{\partial r}\left(\frac{i'(\tau)}{r}\right) \\
&= \frac{\sin\theta d\ell}{4\pi v_p}\left(-\frac{i'(\tau)}{r^2} + \frac{1}{r}\frac{\partial i'(\tau)}{\partial r}\right) = \frac{\sin\theta d\ell}{4\pi v_p}\left(-\frac{i'(\tau)}{r^2} - \frac{1}{r}\left(\frac{1}{v_p}\frac{\partial i'(\tau)}{\partial t}\right)\right) \\
&= -\frac{\sin\theta d\ell}{4\pi v_p}\left(\frac{i'(\tau)}{r^2} + \frac{t_p}{r^2}\frac{\partial i'(\tau)}{\partial t}\right) = -\frac{1}{v_p}\frac{\sin\theta}{4\pi r^2}\left(i'(\tau) + t_p\frac{\partial i'(\tau)}{\partial t}\right) \\
&= -\frac{1}{v_p}\left[\frac{\underline{H}_\phi(\tau)}{t_p} + \frac{\partial \underline{H}_\phi(\tau)}{\partial t}\right] = -\frac{\underline{H}_\phi(\tau)}{r} - \frac{1}{v_p}\frac{\partial \underline{H}_\phi(\tau)}{\partial t}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\nabla \times \boldsymbol{E}(\tau) &= Z_m\left(\frac{\underline{H}_\phi(\tau)}{r} + \frac{\partial \underline{H}_\phi(\tau)}{\partial r}\right) \\
&= Z_m\left(\frac{\underline{H}_\phi(\tau)}{r} - \frac{\underline{H}_\phi(\tau)}{r} - \frac{1}{v_p}\frac{\partial \underline{H}_\phi(\tau)}{\partial t}\right) = -\frac{Z_m}{v_p}\frac{\partial \underline{H}_\phi(\tau)}{\partial t}
\end{aligned}$$

However,

$$-\frac{Z_m}{v_p}\frac{\partial \boldsymbol{H}_\phi(\tau)}{\partial t} = -\frac{Z_m}{v_p}\frac{\partial\left(H_\phi(\tau) + \underline{H}_\phi(\tau)\right)}{\partial t}$$

So, for any $i(t)d\ell$

$$\nabla \times \boldsymbol{E}(\tau) \neq -\frac{Z_m}{v_p}\frac{\partial \boldsymbol{H}(\tau)}{\partial t}$$

Lastly, for any $\boldsymbol{A} = (A_r, A_\theta, A_\phi)$,

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

And, for the fields $\boldsymbol{H}(\tau) = (0,\ 0,\ \boldsymbol{H}_\phi(\tau))$ and $\boldsymbol{E}(\tau) = (\boldsymbol{E}_r(\tau),\ \boldsymbol{E}_\theta(\tau),\ 0)$ of any nanocurrent $i(t)d\ell$,

$$\nabla \cdot \boldsymbol{H}(\tau) = \frac{1}{r\sin\theta}\frac{\partial \boldsymbol{H}_\phi(\tau)}{\partial\phi} = 0$$

and

$$\begin{aligned}
\nabla \cdot \boldsymbol{E}(\tau) &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \boldsymbol{E}_r(\tau)) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \boldsymbol{E}_\theta(\tau)) \\
&= 2\frac{\boldsymbol{E}_r(\tau)}{r} + \frac{\partial \boldsymbol{E}_r(\tau)}{\partial r} + \frac{2\cos\theta}{r\sin\theta}\boldsymbol{E}_\theta(\tau) \neq 0
\end{aligned}$$

Thus, $\nabla \cdot \boldsymbol{H}(\tau) = 0$, but $\nabla \cdot \boldsymbol{E}(\tau) \neq 0$. So, *three of Maxwell's four equations are not satisfied by the fields of a nanocurrent* $i(t)d\ell$ *– a true current element.*

Observations and Conclusions

From an engineering point of view, any *nanocurrent* $i(t)d\ell$ is *a basic source* of the *magnetic field*

$$H_\phi(\tau) = \frac{\sin\theta}{4\pi r^2} i(\tau)d\ell \qquad \text{(A/m)}$$

and the *electric field*

$$E_r(\tau) = \frac{Z_m v_p}{4\pi r^2}\frac{i(\tau)d\ell}{v_{pc}} \qquad \text{(V/m)}$$

And, the *time-variation of* $i(t)$ causes the *additional magnetic field*

$$\underline{H}_\phi(\tau) = t_p\frac{\partial H_\phi(\tau)}{\partial\tau} = \frac{\sin\theta}{4\pi r^2} t_p i'(\tau)d\ell \qquad \text{(A/m)}$$

and the *additional electric fields*

$$\underline{E}_\theta(\tau) = Z_m t_p\frac{\partial H_\phi(\tau)}{\partial\tau} = Z_m\frac{\sin\theta}{4\pi r^2} t_p i'(\tau)d\ell \qquad \text{(V/m)}$$

and

$$\underline{E}_r(\tau) = t_p\frac{\partial E_r(\tau)}{\partial\tau} = \frac{Z_m v_p}{4\pi r^2}\frac{t_p i'(\tau)d\ell}{v_{pc}} \qquad \text{(V/m)}$$

Thus, *the total magnetic field of* $i(t)d\ell$ *is*

$$\boldsymbol{H}(\tau) = H_\phi(\tau) + \underline{H}_\phi(\tau) \qquad \text{(A/m)}$$

and, *the total electric field of* $i(t)d\ell$ *is*

$$\boldsymbol{E}(\tau) = E_r(\tau) + \underline{E}_r(\tau) + \underline{E}_\theta(\tau) \qquad \text{(V/m)}$$

These basic field causes and their fields are keys to understandable engineering electromagnetics. Each field description is accurate for all $r > 0$, because all fields are viewed as leaving the midpoint of *i(t)dℓ*, where their causes are centered. And, each field description should be easily understandable, because each field cause is a factor in its field's description.

The Hertzian dipole has been a useful tool for over a century, but it is inaccurate for near-field use, and the onset of nanoscale electronics makes its replacement with the nanocurrent extremely necessary.

Another conclusion resulting from the above is that abstract mathematics should never be used until a good physical understanding of the subject under study has been achieved. That follows, because Maxwell's equations have long been used and taught [3 – 8], but it is clearly shown above that they are not correct for a very common point source, the nanocurrent *i(t)dℓ*.

References

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About the Author

W. Scott Bennett, Ph.D., served as Assistant Professor at Virginia Polytechnic Institute, where he taught electromagnetics and computer design. He later worked at Hewlett-Packard Company where for 16 years he designed computers and made those designs electromagnetically compatible. Since retiring he has worked to rid basic electromagnetics of abstract mathematics and thus make it easier to understand. He can be reached at: w.scottbennett@juno.com.