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Multi-Section Crystal Bandstop Filter Design

By Mark Mell, Crane Aerospace & Electronics; William B. Lurie, Consultant

This article explains a design procedure for crystal filters with a bandstop response, accomplished by adapting a well-known transform from a lowpass prototype

This article describes two extensions of the Holt-and-Gray technique, which yields multi-section narrowband crystal bandpass filters with elliptic filter characteristics in both passbands and stopbands.

The first extension is relatively minor, in that it refers to a computer patch which is used to minimize the spread of crystal motional parameters throughout the various sections. As described it is interactive, but could be made the objective function of an optimization routine if desired.

The second extension has more significance, in that it describes a technique for synthesizing bandstop (band-reject) filters, again with elliptic function characteristics in passbands and stopbands.

The evolution of the bandpass filter design from the Cauer lowpass prototype is described in the Holt and Gray article [1], and summarized here. For convenience, the basic equivalence on which these techniques are based is shown on Figure 1.

First, a lowpass ladder prototype is selected, one which will, when transformed to bandpass, give the desired characteristics with regard to amplitude, phase, or delay. It should be of an even degree, in the topology of a Cauer elliptic function lowpass. The Holt and Gray article describes its transformation, section by section, into equivalent lattice sections, with constant-impedance inverters

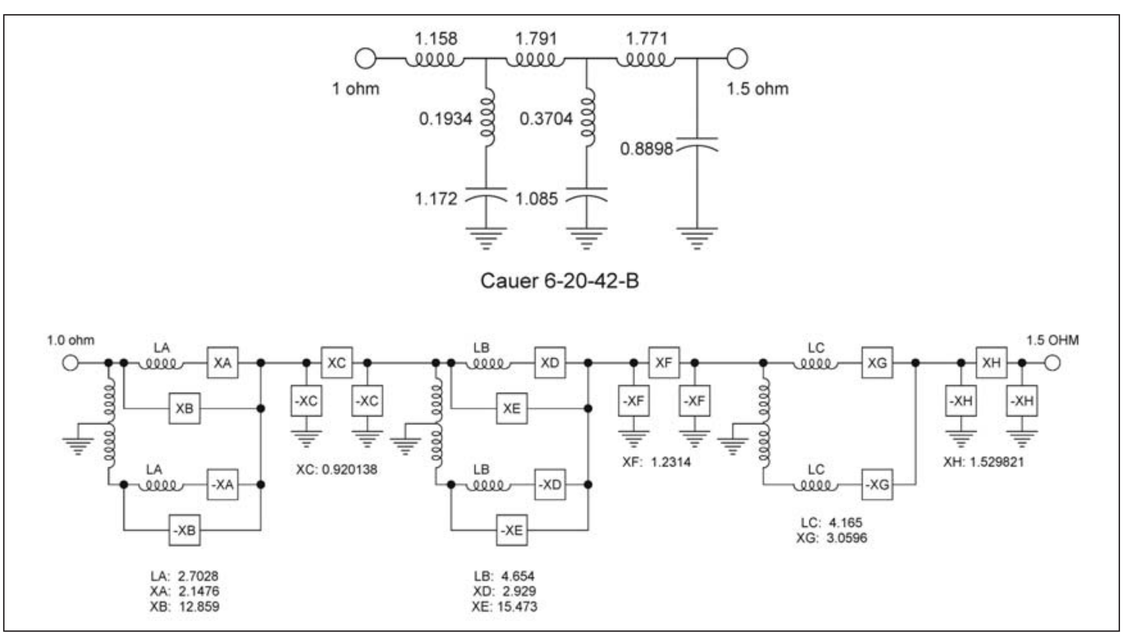

Figure 1 · Holt and Gray lowpass transform (bottom) from an elliptic lowpass prototype.

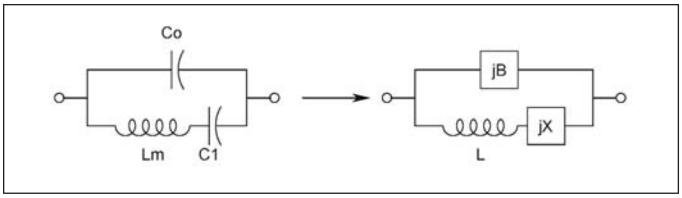

Figure 2 · Approximate bandpass to lowpass transformation of a crystal resonator.

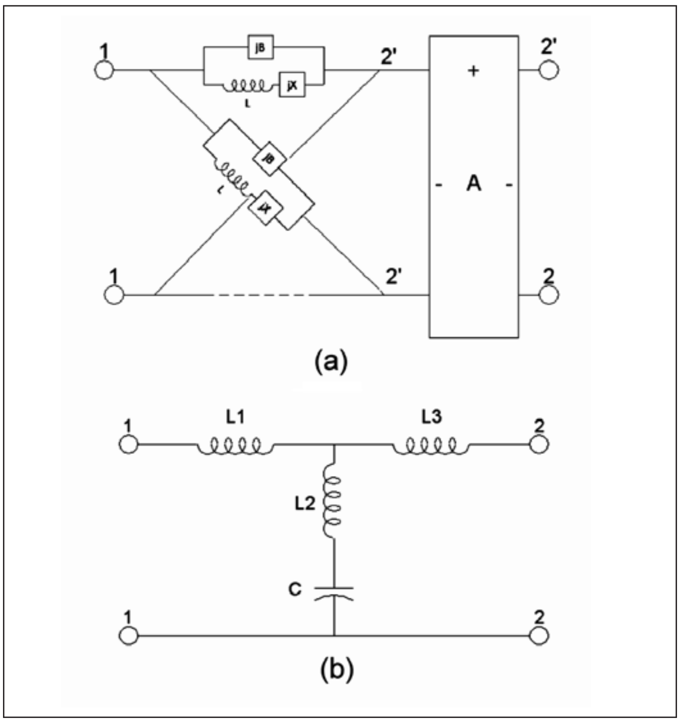

Figure 3 · (a) Second order low-pass section of inductances and frequency independent susceptances. (b) An LC ladder equivalent to (a).

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Table 1 · Parameter selection procedure in the design program.

between sections. The equivalences are shown in Figures 2 and 3, and simple formulas for the equivalences are given in the article. A lowpass to bandpass transformation is then performed on the cascade of lattice sections and impedance inverters, and the impedance inverters are then absorbed into the lattice sections.

The computer program, which we wrote in the APL language, follows Holt and Gray's steps, and its use is shown in the interactive session which follows. The design process has built in the option to make the amplitude response unsymmetrical using a bilinear transformation, as would be desirable for single-sideband applications. It also offers the opportunity to present an equivalent design for fabrication in multi-crystal monolithic form, as converted by S.K. Lu [2].

After an initial pass, the user may examine the crystal motional parameters which are presented separately, to decide if it would be more practical with an equivalent design in which the spread of values would be minimized.

In the Holt and Gray article we see that if "two impedance inverters are impedance scaled by a factor r, this property can be used n–1 times to equate all the impedance inverters of an n-section filter, and one more time for some other purpose, e.g., to equate the inductances of any two sections or to equate source to load conductance." This is accomplished in our present embodiment by manually tweaking the impedance inverter constants in "trial and error" manner, looping back to where those constants were entered in the design process. Minor modifications could be made in the program that would allow the program to serve as the objective function in an optimization routine, but the manual process is simple and takes only a few iterations to make (in the six-crystal example), four of the six crystals identical in motional capacitance, as shown.

Table 1 shows the actual interactive operation of the program; the steps are primarily self-explanatory.

Development of Crystal Bandstop Filter

The low-pass ladder filter to crystal bandpass filter transformation devised by Holt and Gray can readily be extended to crystal bandstop filters. When realized as is customary in a semi-lattice configuration, the balun transformers limit the usable pass-band bandwidth. While this limitation can easily be accommodated in many applications, careful selection of the circuit topology and crystal parameters can increase the bandwidth.

After selecting a low-pass prototype filter that meets the performance objectives and completing the Holt and Gray transform, a low-pass to high-pass transformation is made. The transform is carried out by replacing the complex frequency variable "s" within the transfer function with its inverse. As expected in the corresponding network the frequency variable *j*ω and any associated constant are both inverted. In our case this transform replaces each inductor with a capacitor whose capacitance is equivalent to the inverse of the lowpass inductor value. As is also typical within a low-pass to high-pass transform, all non-frequency dependent constants remain unmodified.

The sixth order Elliptic low-pass filter with a 20% reflection coefficient and a modular angle of 42 degrees was again selected as an example. Since unequal terminations can easily be accommodated in a bandstop configuration the Cauer "B" design was selected. The original normalized low-pass circuit and the Holt and Gray transformed circuit are both shown in Figure 1. Note that for simplicity all constant reactance elements are shown in box form indicating the sign associated with the value.

Figure 4 displays the frequency domain analysis of the transformed circuit. In the analysis the semi-lattice transformers were given an inductance value of infinity thus eliminating any contribution to the response. As expected the results between the original lowpass ladder filter and the transformed filter are identical.

Following the procedure outlined above, each inductor is replaced by a capacitor whose capacitance is equivalent to the inverse of the lowpass inductor value. The transformers used to realize the semi-lattice sections are not part of the transform and therefore remain unchanged. Additionally, note that the constant-reactance elements have no frequency dependence, so they too remain unchanged.

After performing the low-pass to high-pass transform the resulting network is ready to be frequency scaled. Frequency scaling is accomplished by dividing all frequency selective elements by a constant equal to the desired bandwidth, in radians, of both the high-pass and ultimately the band-reject filter. For this example a bandwidth of 10 kHz was chosen. The transformed high-pass network is shown in Figure 5 and the circuit's response is given in Figure 6.

After frequency scaling, the network is ready for the highpass to bandstop transform. The highpass to bandstop transform consists of replacing the complex frequency variable "*s*" within the transfer function with itself plus its inverse. While this often yields an equation that is quite unwieldy, the associated network transformation is simpler to implement. An inductor need only be added across each network capacitor where the inductor's value is that which, along with the capacitor, is resonant at the band-reject filter's center frequency. Continuing with the example the transformed network is shown in Figure 7 and the circuit's response is given in Figure 8.

Although the network of Figure 7 exhibits the desired response, it is

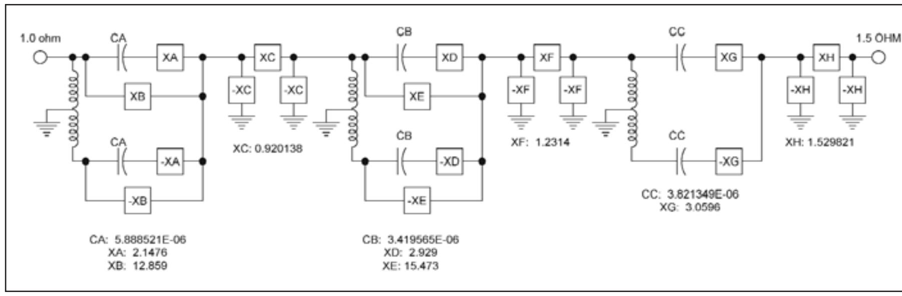

Figure 5 · Lowpass to high-pass transform.

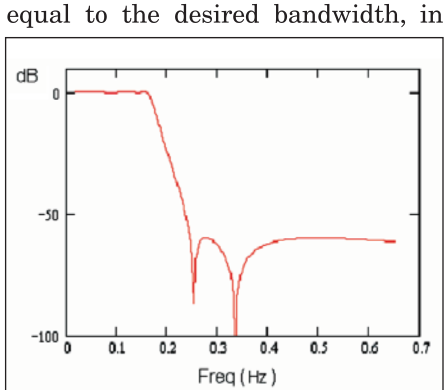

Figure 4 · Frequency response of the lowpass transformed circuit.

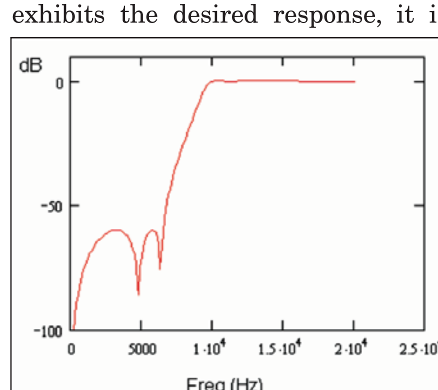

Figure 6 · Frequency response after highpass transformation.

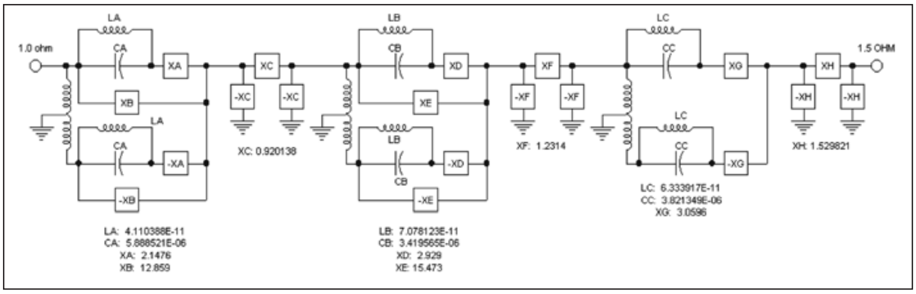

Figure 7 · Initial results of the bandstop transform.

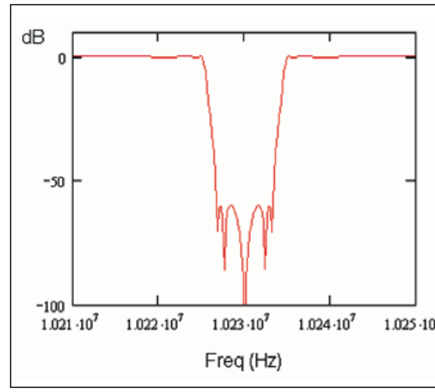

Figure 8 · Bandstop filter frequency response.

completely unrealizable. To yield a circuit that can actually be constructed, we must first eliminate all constant reactance elements. Since the underlying constraint of all crystal filters is that of limited bandwidth, the ratio of frequency selective to constant reactance elements within a given semi-lattice section is extreme.

As a result, minimal inaccuracies are introduced by converting all constant reactance elements to frequency dependent reactive elements within the semi-lattice sections. For simplicity it is convenient to represent all previous constant reactance elements as capacitors with a reactance equivalent to the constant reactance value at the band-reject filter's center frequency. After the conversion the networks can be simplified using the network transformations detailed by Zverev [3]. Figure 9 contains the network after the simplification.

Before proceeding to eliminate the remaining constant reactance elements, a few more transforms are needed. It is always advantageous to reduce the spread of inductance values within a network. This is especially critical when implementing crystals as reactive elements. Each inductance value is associated with relatively expensive tooling in the form of a mask that controls the physical size of the electrode deposited on the quartz blank itself. By eliminating all but one value the cost of realization is considerably reduced and the likelihood of performance degradation associated with the stack up of multiple tolerances is eliminated. To make the inductor values of the first and second semi-lattice sections equal, the first and second impedance inverters are impedance scaled. The scalar constant is simply the square root of the quotient of the second section inductor value divided by the first. As is typical, the impedance inverters are both scaled by this factor while all elements between the impedance inverters are scaled by the factor squared. In this example this step is repeated a second time to scale the third and final semi-lattice section. Figure 10 contains the network after equalizing the inductor values.

With the equalization of the inductance values complete the last impedance inverter can be eliminated by scaling its value to one, modifying the load resistance accordingly and then by removing the impedance inverter and inverting the load. After completing this step it might seem attractive to apply a

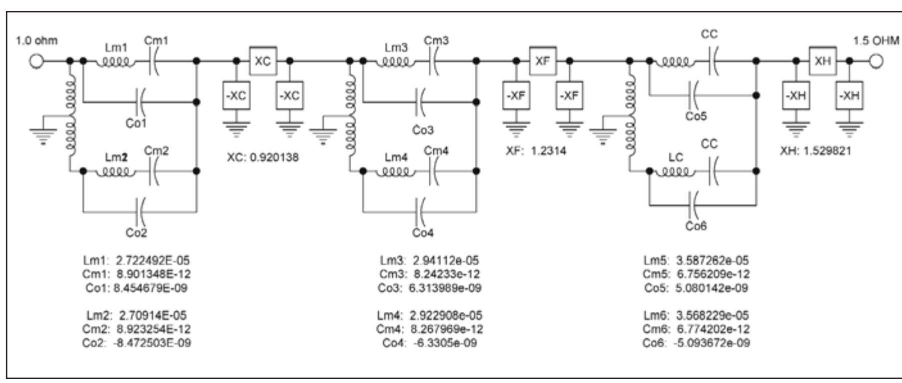

Figure 9 · Bandstop network after simplification.

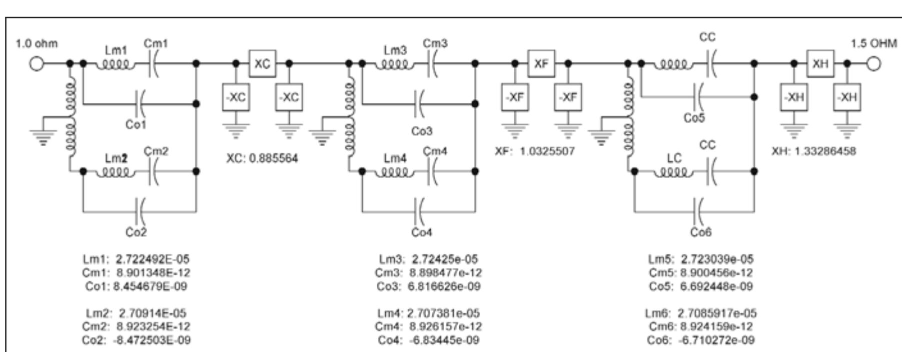

Figure 10 · The network after equalizing the inductor values.

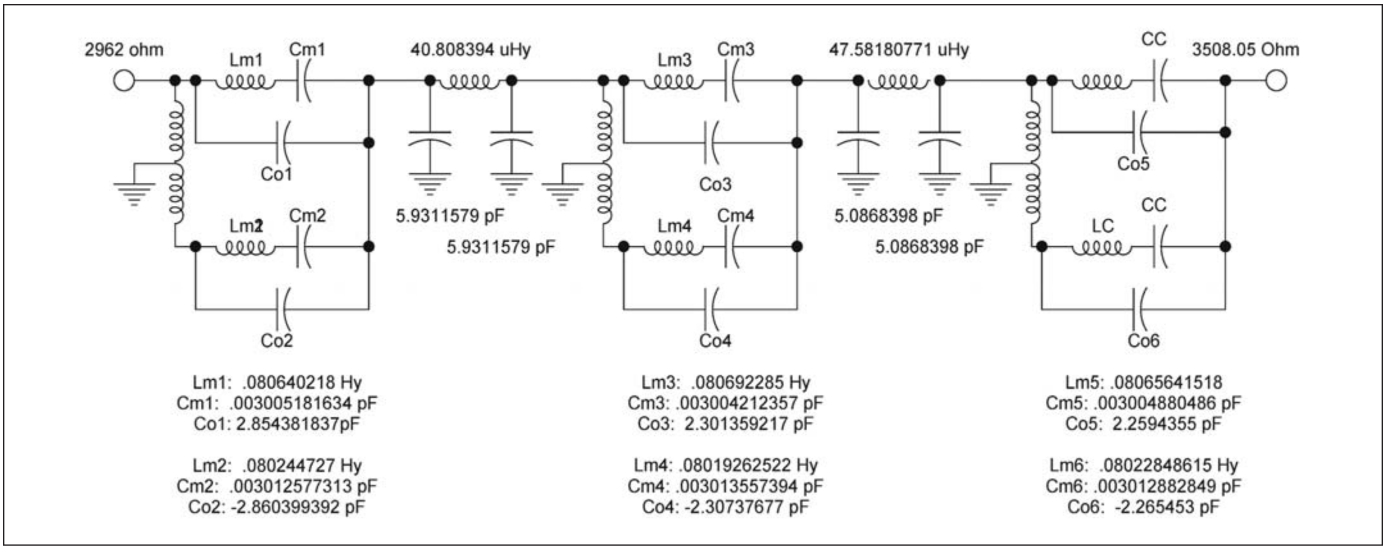

Figure 11 · The final bandstop filter network, after scaling for the desired motional inductance and making impedance inverter substitutions.

Pi to T transformation on the two remaining impedance inverters and, by including complementary reactance elements at both the input and output of the network, pull series reactance elements into the semi-lattice sections. Note the Pi to T transition is needed so the final configuration would have a shunt reactance between the semi-lattice sections. A series element left here is not ultimately helpful. While this configuration would result in a usable filter it would also be reintroducing multiple inductance values and, more importantly, would raise the source and load impedance after implementing the necessary Thévenin equivalents to accommodate the remaining series reactance elements. Through the use of optimization both the ratio of inductances and the source and load impedances can be reduced, but ultimately the bandwidth will suffer. The best approach is to leave all impedance inverters in Pi form.

The next step consists of impedance scaling the circuit to one consistent with the inductance values of the crystals. Care must be taken when selecting this value. Crystal motional inductance values that are too high will limit the overall bandwidth due to the effect of real-world inductors implementing the balun of the semi-lattice section. Motional inductance values that are too low are typically associated with a higher amount of spurious which can produce unwanted very narrow peaks of attenuation within the upper pass-band of the filter. The requirements of the application will dictate to which extreme the designer gravitates.

After impedance scaling the remaining constant reactance elements need to be addressed. Once again the only option is to replace all constant reactance elements with frequency dependent ones. Given this restriction the designer still has considerable flexibility regarding how the substitution is made. The only final limitation is that the sign difference of the reactance elements within a given impedance inverter must be maintained. Specifically if one chooses an inductance as the series element of the impedance inverter then the shunt elements must be capacitive. This configuration is frequently advantageous for filters where greater bandwidth is needed below the notch since the contribution of the impedance inverter far from the pass-band is that of a low-pass filter. In this arrangement the actual lower cutoff will be solely dependent on the value of the balun transformer. Conversely, if more bandwidth is needed above the pass-band, a high-pass circuit could be used instead.

The final filter after scaling the circuit for a motional inductance value of 80 mH and implementing the impedance inverter substitutions is shown in Figure 11. The 80 mH value equates to a crystal fabricated with .062 inch electrodes which, if processed correctly at this frequency, should be nearly spur free.

The remaining negative capacitance values can be accommodated in the usual way by pulling twice their value in from the shunt elements on either side of the semi-lattice sections. Since adequate capacitance already exists at most of these nodes the balun transformers can be self resonant at the notch's center frequency thus optimizing the bandwidth. Balun transformers with a primary inductance of 20 µH realized using double aperture cores are close to self resonance at the filter's center frequency. In this example, when including the transformers in the analysis a percentage bandwidth of approximately 48% is achieved.

In summary, two extensions of the Holt-and-Gray technique have been described. The first a mechanism for reducing the ratio of the motional inductance of the crystal branch arms aiding in their realization. In the second a crystal notch filter with a reasonably wide ultimate pass-band was shown achievable by careful selection of the crystal's motional parameters and by implementing the impedance inverters within the circuit as either low-pass or high-pass structures. Any attempt to absorb the impedance inverters within the semi-lattice sections will ultimately reduce the usable bandwidth.

References

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Author Information

William B. Lurie graduated from Yale University at the age of 18 with degrees in Mathematics and Physics, completing his M.A. in Education the following year. Upon graduating Bill worked for the Navy Department as a physicist, followed by work at the GPL Division of General Precision Inc., where he led the development of major portions of airborne Doppler radar navigation systems for aircraft. In 1961, Bill became VP of Engineering at Burnell & Company, a company primarily focused on the design and development of crystal and lumped constant filters. Since 1969, Bill has worked as an independent consultant assisting with design and synthesis problems. Today, Bill is technically retired, spending more hours on the golf course than resolving technical problems, but continues to be a life long mentor to many individuals in the industry. He is a Life Senior Member of IEEE. Bill can be reached at billurie@ieee.org

Mark P. Mell has worked in the design and synthesis of crystal and lumped constant filters since graduating from LTI in 1980. His employment history includes 6 years with OPT Industries as their principal filter development engineer, followed by employment at Microsonics, which is now a division of Crane Aerospace & Electronics, where he is presently the lead design engineer for the Beverly, MA site. His most recent work includes the design of microwave and RF filters, phase shifters, frequency discriminators, digital attenuators, and mixer based products. He can be reached at: Mark.Mell@crane-eg.com