Advanced Characterization of Non-Uniform Passive Devices

By Dr. Hatem Akel
BreconRidge

This article describes advanced de-embedding techniques for back-to-back configurations which enable measurement of devices that do not have uniform 50-ohm inputs and/or outputs

Non-uniform passive RF devices are structures that have non-identical ports and/or ports that function at different frequencies. For example, a device with a microstrip input port and an SMA output port, is a non-uniform device. Mixers, multipliers and dividers are other examples of non-uniform devices due to the fact that their ports operate at different frequencies. Measuring non-uniform devices is a challenging mission. Indeed, standard calibration kits and procedures cannot be used with non-uniform devices. Calibration can only be performed if, and only if, the ports of the device are identical and function at the same frequency.

Designers have to use additional components like transformers or transitions, combining them with the original device under test (DUT) to create a new device with uniform ports. After calibration, the measured data represents the performance of the DUT plus the additional component(s). If the scattering matrix of the additional component(s) is known, then one can easily de-embed the additional component(s) from the total to obtain the performance of the DUT.

Back-to-back measurement was proposed as a way to overcome the non-uniformity problem [1]. Back-to-back configuration enables the user to eliminate one of the ports, by creating a complex device with the same port type and frequency (Figure 1). Consequently traditional calibration and measurements can be applied directly without any concerns. RF designers can choose two devices of the same kind to connect them back to back, or can have two totally different devices, as long as the ports and their operating frequencies are the same in both devices.

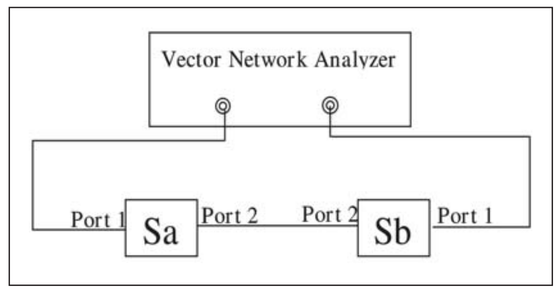

Figure 1 · Back-to-back cascading measurements.

The Ill-Conditioned Issue of the Back-to-Back Measurements

Cascading two devices back-to-back leads to a set of three complex equations with six complex unknowns; Sa_{11}, Sa_{12}, Sa_{22}, Sb_{11}, Sb_{12}, Sb_{22}. That means the problem is ill-conditioned and cannot be solved. If both devices were perfectly identical, which is impossible, then the problem becomes solving two equations with three unknowns. This is also an ill-conditioned problem.

Cascading three devices back to back: #1 with #2, #1 with #3, and #2 with #3, generates nine complex equations with nine unknowns. Although, the number of unknowns and the number of equations are the same, the problem is still ill-conditioned because there is one redundancy in the equations. Even if one uses four or more devices, the problem is still there: one redundancy. This redundancy means that

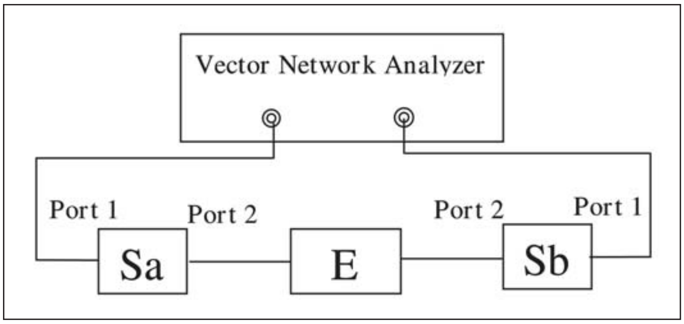

Figure 2 · Back-to-back cascading measurements with an additional component in between.

Cascading Three Devices Back to Back, with Additional Component in Between	Generating Sd by Cascading Sa with E for the New Setup
Sa E Sb	Sd Sb
Sa E Sc	Sd Sc
Sb E Sc	Sb E Sc

Table 1 · Before and after combining *Sa* with the additional component, *E*.

one of the scattering elements of any of the devices must be known ahead to be able to calculate the remaining ones.

Solving the Ill-Conditioned Issue

To overcome the ill-condition problem, we proposed to make use of the fact that for any two-port passive devices: the amplitudes of the return losses are very close. First, random values are assigned, for example to Sb_{22}, while the rest of the elements are calculated. A set of formulae (Eq.4) was derived to calculate all scattering elements of the three devices under test, assuming that Sb_{22} is known. The criterion is to look for the value of Sb_{22} that minimizes the three following equations respectively:

$$\begin{aligned}&\mathrm{ABS}\left(\left|\mathrm{Sa}_{11}\right|-\left|\mathrm{Sa}_{22}\right|\right)<\text{tolerance}\\&\mathrm{ABS}\left(\left|\mathrm{Sb}_{11}\right|-\left|\mathrm{Sb}_{22}\right|\right)<\text{tolerance}\\&\mathrm{ABS}\left(\left|\mathrm{Sc}_{11}\right|-\left|\mathrm{Sc}_{22}\right|\right)<\text{tolerance}\end{aligned}\tag{1}$$

Sb_{22} random values can be obtained by splitting the smith chart complex plane into a 200 × 200 grid, assign Sb_{22} one complex value at a time, evaluate all the scattering elements for every complex point, then choose the one that minimized the maximum of the three differences in Eq.1 (minimizing the maximum was found more reliable than minimizing the summation of the differences). If a more accurate value is needed, create a 20 × 20 grid around the optimum point, with 1/10 scale and repeat the calculations. The process can be repeated for as many decimals as needed.

De-Embedding In-Between Additional Component(s)

In many situations, an additional component(s) is needed in between the DUTs connected back-to-back. Designers would like to de-embed the effect of this component and obtain the scattering matrices of the DUTs (Fig. 2). For example, if the ports we would like to connect are both female SMA type of ports, then one needs to use a male-to-male SMA cable to make the connection. To obtain an accurate scattering matrix of the DUT, the male-to-male transition should be de-embedded from the measured data. In the rest of the paper, the scattering matrix of the additional component is referred to as the E matrix.

The assumption made is that this additional component can be measured and consequently its scattering matrix is known. To simplify the analysis and the derivations, one can combine the first device with the additional one, to form a new component with a scattering matrix labeled Sd. This will enable us to use many of the formulae given in [1]. Once Sd is evaluated, then the additional component is de-embedded from Sd, to extract Sa, which the scattering matrix of the first device (Table 1).

The difference between the new configuration and the one in [1] is in the third set, while the first two sets are the same. The cascading formulae for the first set are:

$$\begin{aligned}&A_{11}=\left(Sd_{11}+Sd_{21}Sb_{22}W\,Sb_{21}\right)\\&A_{21}=Sb_{21}W\,Sd_{21}\\&A_{22}=Sb_{21}W\,Sd_{22}Sb_{21}+Sb_{11}\\&\textit{where}\\&W=\left(1.0-Sd_{22}Sb_{22}\right)^{-1}\end{aligned}\tag{2}$$

Where the A-matrix is the result of cascading Sd with Sb. For the second set, Sb is replaced with Sc to obtain the B-matrix. Notice that we are again considering the cases where $Sa_{21} = Sa_{12}$, $Sb_{21} = Sb_{12}$ and $Sc_{21} = Sc_{12}$.

For the third set it is:

$$\begin{aligned}&D_{11}=Sb_{11}+\left(Sb_{21}E_{11}/E_{21}+D_{21}Sc_{22}/Sc_{21}\right)\left(\frac{E_{21}Sb_{21}}{1-Sb_{22}E_{11}}\right)\\&D_{21}=\frac{E_{21}Sb_{21}Sc_{21}}{\left(1-E_{22}Sc_{22}\right)\left(1-Sb_{22}E_{11}\right)-E_{21}^{2}Sb_{22}Sc_{22}}\\&D_{22}=Sc_{11}+\left(E_{22}\left(1-Sb_{22}E_{11}\right)+E_{21}^{2}Sb_{22}\right)\left(\frac{D_{21}Sc_{21}}{Sb_{21}E_{21}}\right)\end{aligned}\tag{3}$$

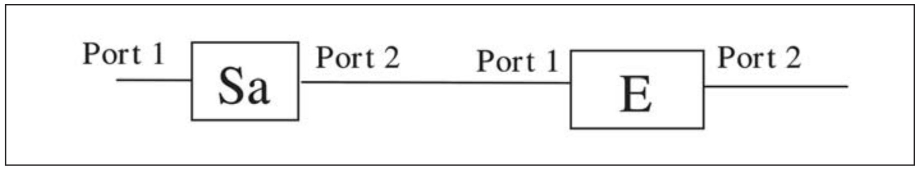

Figure 3 · Combining *Sa* with *E* to form a new component.

where the D-matrix is the result of cascading Sb with E and with Sc.

Solving for the Scattering Matrices

If Sb_{22} is known, then the following formulae (Eq.4) can be used to derive the rest of the elements Sd_{11}, Sd_{21}, Sd_{22}, Sb_{11}, Sb_{21}, Sc_{11}, Sc_{21}, Sc_{22}

$$\begin{aligned}
&a\,Sc_{22}^2 + b\,Sc_{22} + c = 0 \\
&where \\
&a = \alpha_5 * \alpha_3 \\
&b = \alpha_4 * \alpha_3 + \alpha_5 * \alpha_2 - \alpha_1 * \alpha_7 \\
&c = \alpha_4 * \alpha_2 - \alpha_1 * \alpha_6 \\
&and \\
&\alpha_1 = (B_{22} - D_{22}) + \frac{D_{21}B_{21}}{E_{21}A_{22}}\left(E_{22} - \left(E_{22}E_{11} - E_{21}^2\right)Sb_{22}\right) \\
&\alpha_2 = (B_{22} - D_{22})Sb_{22} + \frac{B_{21}^2}{A_{11} - B_{11}}Sb_{22} \\
&\alpha_3 = \frac{-B_{21}^2}{A_{11} - B_{11}} + \frac{D_{21}B_{21}}{E_{21}A_{21}}\left(E_{22} - \left(E_{22}E_{11} - E_{21}^2\right)Sb_{22}\right) \\
&\alpha_4 = (A_{22} - D_{11})(1 - E_{11}Sb_{22}) + \frac{E_{11}A_{21}^2}{A_{11} - B_{11}}Sb_{22} \\
&\alpha_5 = \frac{-E_{11}A_{21}^2}{A_{11} - B_{11}} + \frac{E_{21}D_{21}A_{21}}{B_{21}} \\
&\alpha_6 = (A_{22} - D_{11}) + \frac{A_{21}^2}{A_{11} - B_{11}}(1 - E_{11}Sb_{22})Sb_{22} + \frac{E_{11}A_{21}^2}{A_{11} - B_{11}}Sb_{22}^2 \\
&\alpha_7 = (A_{22} - D_{11})(1 - E_{11}Sb_{22}) - \frac{A_{21}^2}{A_{11} - B_{11}}(1 - E_{11}Sb_{22}) \\
&\quad - \frac{E_{11}A_{21}^2}{A_{11} - B_{11}}Sb_{22} + \frac{E_{21}D_{21}A_{21}}{B_{21}}Sb_{22}
\end{aligned} \tag{1}$$

$$Sd_{22} = \frac{\alpha_1}{\alpha_2 + \alpha_3 Sc_{22}} \tag{2}$$

$$Sd_{21} = \sqrt{\frac{(1 - Sd_{22}Sb_{22})(1 - Sd_{22}Sc_{22})(A_{11} - B_{11})}{(Sb_{22} - Sc_{22})}} \tag{3}$$

$$Sb_{21} = A_{21}(1 - Sd_{22}Sb_{22}) / Sd_{21} \tag{4}$$

$$Sc_{21} = B_{21}(1 - Sd_{22}Sc_{22}) / Sd_{21} \tag{5}$$

$$Sd_{11} = A_{11} - Sd_{21}Sb_{22}A_{21} / Sb_{21} \quad [6]$$

$$Sb_{11} = A_{22} - Sb_{21}Sd_{22}A_{21} / Sd_{21} \quad [7]$$

$$Sc_{11} = B_{22} - Sc_{21}Sd_{22}B_{21} / Sd_{21} \quad [8]$$

The optimum solution is the one that minimizes the maximum of Eq. 1 formulae. The analysis proved that the solution to any given A, B and D matrices is unique, and that means that the approach presented in this paper is consistent.

This approach was tested for many combinations where all or some of the samples were too lossy, and all or some of the samples had very bad return losses. Samples with known scattering matrices were cascaded (sample #1 with sample #2, sample #2 with sample #3, and sample #1 with sample #3), and the results were fed to the program to extract the original scattering matrices of the three samples. In all cases the results were very close to the exact answers.

Special Case

If there is no additional component, then the problem is similar to the case described in [1]. In this case, the E matrix is replaced with the following matrix:

$$E_{ideal} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (5)$$

That is, the insertion is complete without any delay and there is no return loss effect.

De-embedding Formulae

The analysis given in the previous section demonstrates how Sd, Sb and Sc scattering matrices are calculated from the measured scattering matrices; A, B, and D. Sd is the combination of Sa with the additional component, and it is Sa that needs to be evaluated. Sa is obtained by de-embedding the E-matrix from Sd (Fig. 3).

Assuming that port#2 of Sa that is connected to port#1 of the additional component, E, then Sd is equal to:

$$\begin{aligned} Sd_{11} &= (Sa_{11} + Sa_{12}E_{11}W\,E_{21}) \\ Sd_{12} &= Sa_{12}(Sb_{11}W\,Sa_{22} + 1.0)E_{12} \\ Sd_{21} &= E_{21}W\,Sa_{21} \\ Sd_{22} &= E_{21}W\,Sa_{22}E_{12} + E_{22} \end{aligned} \quad (6)$$

where,

$W = (1.0 - Sa_{22}E_{11})^{-1}$

Rearranging the variables, it can be shown that if E and Sd are known, then the Sa matrix can be calculated using the following set of formulae:

$$\begin{aligned}
&Sa_{12} = Sd_{12}E_{12}^{-1}\left(E_{11}K + 1.0\right)^{-1} \\
&where \\
&K = E_{21}^{-1}\left(Sd_{22} - E_{22}\right)E_{21}^{-1} \\
&Sa_{11} = Sd_{11} - Sa_{12}E_{11}L \\
&where \\
&L = E_{21}^{-1}Sd_{21} \\
&Sa_{22} = K\left(KE_{11} + 1.0\right)^{-1} \\
&Sa_{21} = Sd_{21}\left(1.0 - Sa_{22}Sd_{11}\right)Sd_{21}^{-1}
\end{aligned} \tag{7}$$

Once the scattering matrices of the three samples are known, one can use anyone of them to measure other devices or samples. This is equivalent to saying that the derived scattering matrix is now part of the calibration kit for other measurements. For example, if there are tens of samples of the same kind to be measured, then one can do the following:

- Measure the additional component, if there is one
- Measure three samples using back-to-back setup, (with or without the additional component)
- Extract the scattering matrices of each of the three devices,
- Cascade device # 1 with any of the other devices; 4th, 5th, …, (with or without the additional component)
- And each time, de-embed the scattering matrix of device #1 and the additional component(s) from the total to extract the scattering matrix of the other device.

Conclusion

In this paper we demonstrated that there is a way to measure a device with two different ports and/or frequency of operation, by simply using three-device back-to-back measurements. The technique shown here presents the case when an additional component, with known scattering matrix, is inserted in between to perform the back-to-back connection. The analysis provided in this paper is applicable to cases where the insertion losses are equal (amplitude and phase) for any of the three devices used in the back-to-back measurements, i.e., $Sa_{21} = Sa_{12}$, $Sb_{21} = Sb_{12}$ and $Sc_{21} = Sc_{12}$.

References

1. H. Akel, "Characterization of non-uniform devices using back-to-back measurements," *RF Design,* March, 2007.

Author Information

Dr. Hatem Akel is a Senior RF Engineer At Breconridge, Ottawa, Ontario, Canada. He received the Ph.D from McGill University in 2000 and previously worked at SPAR Aerospace (Satellite Antenna Department) and Nortel Networks (High Speed RF/Optical packaging). He canbe reached by telephone at 613-886-6840 or by e-mail at: hatem_akel@breconridge.com