Is the 5th Harmonic Still Useful for Predicting Data Signal Bandwidth?

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Rather than use the simple rule of thumb of including the 5th harmonic, we should use rise/fall time to detemine the bandwidth required to measure a high-speed digital signal

As we attempt to push data faster and faster through serial interfaces, bit rates get larger and bit periods get smaller. A common assumption made is that as the bit rate increases, the signal contains more energy at higher frequencies and, as a result, higher bandwidth is required for the test equipment such as oscilloscopes used to measure the signals. While this can be true it is not always the case, as we shall see. The 5th harmonic of a data signal (or 2.5 times the bit rate for binary, NRZ data) is often used as a guide for selecting the required bandwidth for test equipment. In reality, the 5th harmonic is a very poor predictor of essential frequency content and in fact has little relationship to critical components of a real data signal. A much better predictor of the necessary measurement bandwidth are the rise/fall times of the system.

An increase in measurement bandwidth almost always comes at a price beyond economic—but also in the form of increased noise and distortion. The increased noise not only impacts amplitude measurements but it will also impact the accuracy of timing measurements—sometimes defeating the purpose of the extra bandwidth in the first place. The best compromise is to use a measurement system with just enough bandwidth to measure the signal accurately while minimizing the extra noise introduced by the measurement system.

First of all, let's think about a perfect square wave. Imagine that we could create a clock with infinitely fast edges. As we know, a perfect square wave can be represented as the sum of an infinite number of sine waves. These sinusoids have frequencies that are at odd multiples of the fundamental frequency of the clock. For example, a perfect 1 GHz clock signal has a fundamental frequency of 1 GHz and contains an infinite series of sinusoids of frequency 3, 5, 7, 9, 11 … GHz. These are referred to as harmonics of the fundamental signal since they are located at (odd) integer multiples of the fundamental frequency. The amplitudes of the harmonics in this perfect clock are defined by the relationship: Amp = abs(sqrt(2)*SquareWaveAmp*sin(pi*harmonic.number/2)/pi*harmonic.number)

So for a perfect clock of peak-to-peak amplitude 1V, the peak-peak voltages of the fundamental and the first three harmonics would be:

Harmonic Number	1 (fundamental)	3rd	5th	7th
Amplitude	0.45V	0.15V	0.09V	0.064V

If we wanted to measure this clock with zero rise/fall times then obviously the more bandwidth our measurement instrument has, the more accurate the measurement. As we can see, the amplitude of the 5th harmonic is 9% of the amplitude of the resultant signal (0.09V/1V).

Of course, real signals have finite rise/fall times. Correspondingly they don't have an infinite number of harmonics. In Figures 1-3, we created a 0.5Vp-p 500 MHz clock with 3 different edge speeds using commonly available transition-time converters. The signals

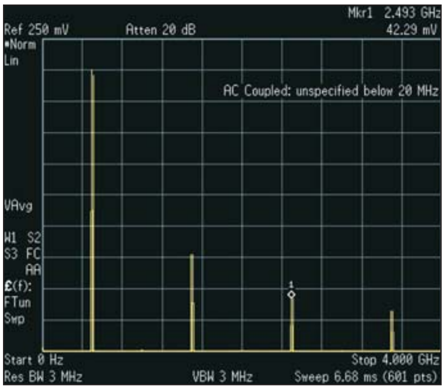

Figure 1 · 500 MHz clock with 9 ps rise/fall time (20-80%); 5th harmonic amplitude = 8.6%.

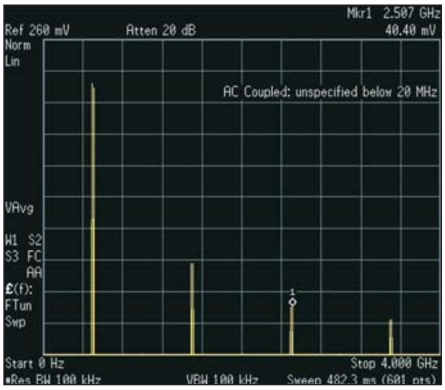

Figure 2 · 500 MHz clock with 35 ps rise/fall time (20-80%); 5th harmonic amplitude = 8.1%.

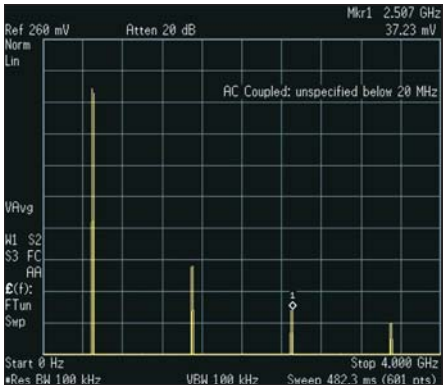

Figure 3 · 500 MHz clock with 47 ps (20-80%) rise/fall time ; 5th harmonic amplitude = 7.4%.

were measured with a 26.5 GHz RF spectrum analyzer to analyze the amplitude of the 5th harmonic.

With a perfect edge, the harmonic amplitude should have been 0.045V (9%), but as the rise/fall time increases, the amplitude of the harmonics decreases.

If we now look at a much faster bit rate of 3 GHz (still 0.5Vp-p) with the same rise/fall times we see that the 5th harmonic has a much smaller relative importance (Figures 4-6). The 5th harmonic of a 3 GHz clock pattern with 47 ps rise/fall times is only 0.3% of the amplitude of the resulting signal.

As we can see, the relative importance of the harmonics depends both on the bit rate and the edge speeds of the signal.

So far we have looked only at clock signals. Let's see what effect changing the data content of a signal has on the spectral content of signals. We took the same signal 0.5Vp-p signal source at 1 Gb/s, 9 ps edge speed, and changed the pattern to the following patterns commonly used in the testing of high speed serial interfaces:

1. PCI Express Compliance Pattern (40 bits long)
2. K28-5 (20 bit long 8b/10b encoded pattern)
3. PRBS $2^{7}-1$ (127 bits long)

in Figures 7-9, the first thing we notice is that there are no clear harmonics at multiples of the fundamental frequency of the signal (bit rate/2). There is plenty of spectral content, but it is spread out at many different frequencies. If we look at the amplitude of the content at the 5th harmonic frequency (in this case 2.5 GHz) we see that even though the edge speed is very fast (9 ps 20-80%) the amplitudes are quite small compared to the amplitude of the resultant signal. We also notice that, with more spectral content, the amplitudes of each spectral tone are lower. This is because the energy in the signal is being distributed among different frequencies. It is clearly not enough to simply look at the amplitude of the spectral content at the 5th harmonic frequency. We must take into account the energy at all frequencies to decide how much bandwidth we need in order to accurately measure a signal.

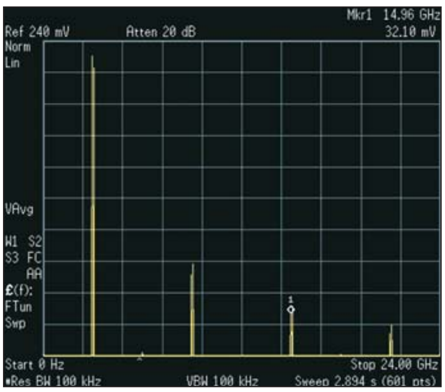

Figure 4 · 3 GHz clock with 9 ps (20-80%) rise/fall time; 5th harmonic amplitude = 6.4%.

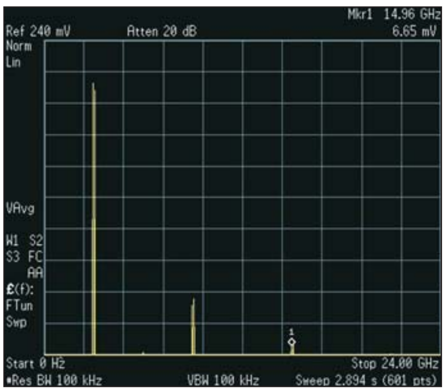

Figure 5 · 3 GHz clock with 35 ps (20-80%) rise/fall time; 5th harmonic amplitude = 1.3%.

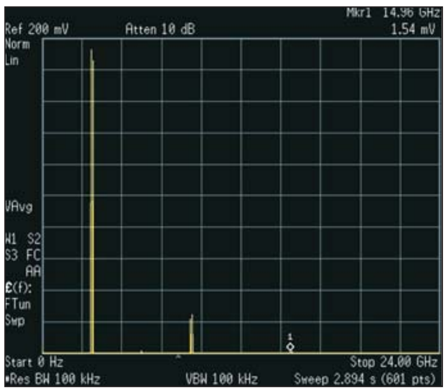

Figure 6 · 3 GHz clock with 47 ps (20-80%) rise/fall time; 5th harmonic amplitude = 0.3%.

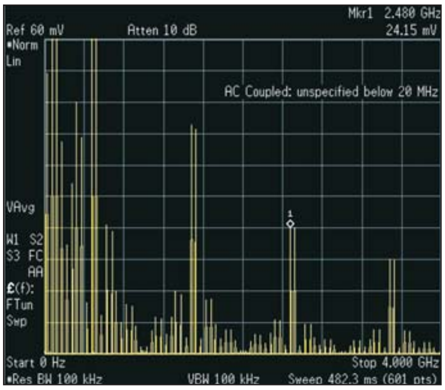

Figure 7 · PCIe Compliance Pattern; 5th harmonic amplitude = 4.8%.

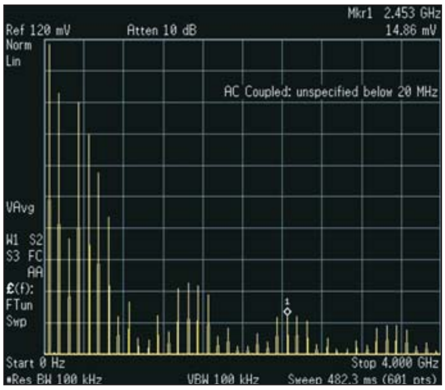

Figure 8 · K28-5 encoded pattern; 5th harmonic amplitude = 3.0%.

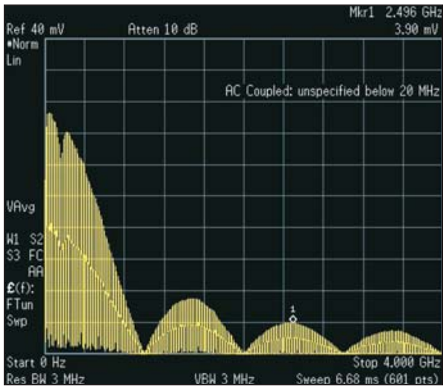

Figure 9 · PRBS $2^7 - 1$; 5th harmonic amplitude = 0.8%.

Another way to quantify how much bandwidth is required to capture the necessary frequency content of a signal is to integrate the spectral content of a signal until we reach a certain percentage of the total signal power (say 99.9%). This method ensures that we include all the spectral content that is of amplitude large enough to significantly affect the signal.

The graphs in Figures 10 and 11 compare the amount of bandwidth (in GHz) required to contain 99.9% of the signal power of a clock pattern and a $2^7 - 1$ PRBS pattern. Both simulated data and live signal measurements are compared at many different combinations of bit rate and edge speed. The edge speeds in the simulation were created using a sum of cosines filter with linear phase. Measurements of the live signal were produced with an Agilent N4903A J-BERT and cascaded Picosecond Pulse Labs transition time converters measured with a 26.5 GHz E4440A performance spectrum analyzer.

The rise time is clearly the dominant factor in determining the amount of bandwidth the signal requires. Indeed, the data even shows that in some cases a higher bit rate requires less bandwidth than a lower bit rate for a given rise time. Note that if we analyze the data closely we see that in most cases the clock pattern requires the most bandwidth for a given bit rate or rise time. Intuitively this makes sense, since the clock is switching states more often than the data pattern and thus contains more high frequency energy

In the real world it is not practical to measure each signal with a wide-band spectrum analyzer before deciding which oscilloscope to use. Although there is no easy way to predict exactly how much bandwidth is required for a given data pattern, bit-rate and rise time, we can use a conservative approximation: the bandwidth required to capture 99.9% of a signal's power is ~0.56/RiseTime. The choice of 99.9% signal power is arbitrary, but it can be shown that for a flat-response real-time oscilloscope, a measurement bandwidth equal or greater to 0.56/RiseTime will deliver rise/fall time measurement accuracy of 3% or better.

The Effect of Noise on Timing Measurements

As we discussed in the introduction, more measurement bandwidth usually means more instrument

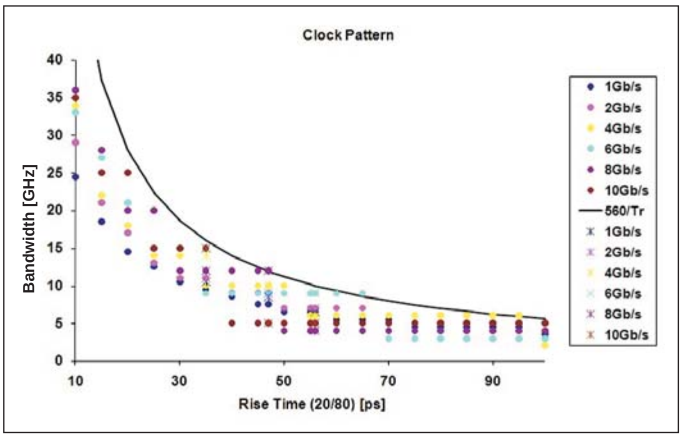

Figure 10 · Bandwidth required for a clock signal at various combinations of bit rate and edge speed.

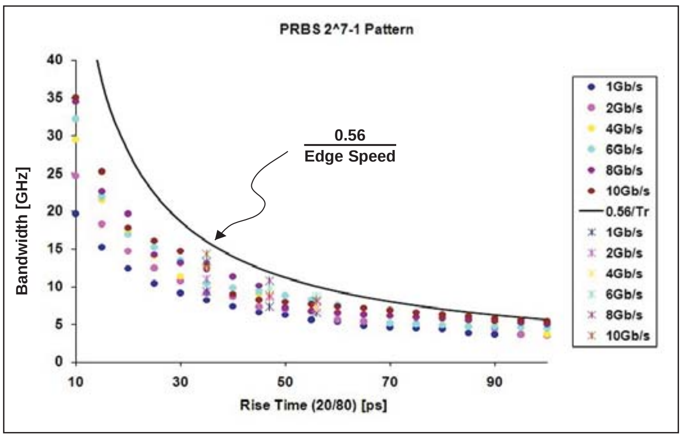

Figure 11 · Bandwidth required for a $2^7 - 1$ PRBS pattern at various combinations of bit rate and edge speed.

Figure 12 · Digital signal without added noise.

Figure 13 · Digital signal with added noise.

noise and noise impacts timing measurements.

In Figures 12 and 13 the effect of additional noise on a signal is illustrated. The signal source is the same in both cases but in Figure 13, noise has been added to the signal.

An Agilent DSO91304A real-time oscilloscope is used to measure the rise time of the signal and its standard deviation. Although the nominal rise time does not change much, the standard deviation and range of the measurement almost doubles.

The white line shows the nominal edge speed, the red line shows an apparent decrease in rise time due to noise, and the green line shows an apparent increase in rise time due to the noise. The slower the edge, the greater the impact of the noise.

If we assume that the instrument noise is mainly Gaussian, independent of signal amplitude, and the slew rate of the signal is similar at both the 20% and 80% locations on the edge, we can approximate that the standard deviation of the 20-80% rise/fall time (σ_{Tr}) is:

$$\sigma_{Tr} > \frac{\sqrt{2} * \sigma_n * Tr}{0.6 * A}$$

where σ_n is the standard deviation of the instrument noise, Tr is the edge speed and A is the nominal signal amplitude. The √2 factor comes from the fact that we are combining the noise on 2 locations on the edge, and the 0.6*A from the choice of 20-80% signal amplitude as our rise time.

For comparison with the bandwidth requirement of 3% accuracy, a 3% standard deviation of the rise/fall time would be created purely by instrument noise when the instrument noise is:

$$\sigma_n = \frac{A}{80}$$

Thus, when measuring a 0.5V p-p signal for example, instrument rms noise of 6.25 mV would introduce a measured rms error in the rise time of approximately 3%.

The actual variation created by the instrument noise could be much higher if the instrument noise is not constant with signal amplitude.

Many other instrument effects will combine to produce measured variation of rise/fall times such as jitter, interleaving errors, etc., so it is very important to minimize the additional effect of instrument noise to avoid using up valuable measurement margin.

Conclusion

We have seen that the 5th harmonic is a very poor predictor of required measurement bandwidth. Instead, rise/fall times can predict required bandwidth much more accurately through the relationship:

$$BW = 0.56/\text{RiseTime}$$

We must also remember that it is not just bandwidth that determines the quality of a timing measurement. Instrument noise degrades the accuracy of all measurements, and so there is little benefit to high measurement bandwidth without also having low noise.

If a signal is repetitive in nature, then an equivalent time sampling oscilloscope such as the Agilent 86100C can be used. This allows for the simultaneous combination of extremely high bandwidth, low noise and ultra-low jitter.

About the Author

Mark Johnson is an Application Engineer in Austin, TX focused on High Speed Digital, Signal Integrity and Optical applications. Mark has an MPhys degree from Lancaster University in England. After working for the High Energy Magnet Research Lab at Texas A&M University for two years, Mark joined Agilent Technologies as an Optical Applications Engineer, covering fiberoptic, digital and telecomm test.