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Inverse Modeling Estimation Algorithm for Multi-Carrier High Power Amplifiers

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This article describes the mathematical basis of a technique for digital pre-distortion using periodic rather than real-time sampling of the system output.

Many predistortion schemes use a memory-less approach [1, 2], but with broadband systems, where the bandwidth is typically larger than five MHz, the PA typically can no longer be considered a memory-less system. With this, the inverse transfer function of the PA—which describes the forward transfer function of the required pre-distorter function—can no longer be obtained using memory-less approaches. The reason for this is that, for any given input envelope value, there are an infinite number of possible values for the inverse function.

To overcome this, the PA output and the pre-distorter can be modeled as the summation of a number of non-linear P^{th} order polynomials operating on the current sample, as well as other past input samples [3]. For the PA, this is given by,

$$y(n) = \sum_{m=0}^{m=M} x(n-m) \sum_{k=1}^{k=p} b_{mk} \left| x(n-m) \right|^{k-1} \qquad (1)$$

where the value of M is the memory depth, or number of taps, P is the polynomial order, and x and y are the input and output baseband samples of the PA, respectively. When memory is negligible, that is $M = 0$, the model (1) reduces to the simple memory-less PA polynomial model given by,

$$y(n) = x(n) \sum_{k=1}^{k=p} b_k \left| x(n) \right|^{k-1}$$

or,

$$x(n)\left[b_1 + b_2 \left| x(n) \right| + b_3 \left| x(n) \right|^2 + \ldots b_P \left| x(n) \right|^{P-1} \right]$$

To perform pre-distortion, using a pre-distorter non-linear memory based model, the following is used,

$$x_{PD}(n) = \sum_{m=0}^{m=L} x(n-m) \sum_{k=1}^{k=K} a_{mk} \left| x(n-m) \right|^{k-1} \qquad (2)$$

where $x_{PD}(n)$ is the pre-distorter output, L is the memory, and K is the polynomial order. Notice here that the pre-distorter Memory depth and polynomial order practically can not be equal to M and P in eq. (1), however, their values are chosen such that the inverse function of eq. (1) implemented in the pre-distorter is a good approximation, which is limited by the programmable circuits used to implement eq. (2) in the digital domain.

Offline PA Inverse Model Estimation

Figure 1 illustrates the hardware implementation of the principle of indirect linearization based on eqs. (1) and (2), or what is also known as the *offline modeling technique* that is used for extracting the pre-distorter coefficients a_{mk} in eq. (2) which represent the PA inverse transfer function [4, 5].

Here, the inverse modeling is done by feeding the PA baseband output complex samples (not input) as inputs to the offline model eq. (2) that is forced iteratively to mimic the inverse of the PA non-linear and memory behaviour model of eq. (1). Due to the fact that the PA dynamics change very slowly over time, this process is not required to be implemented in

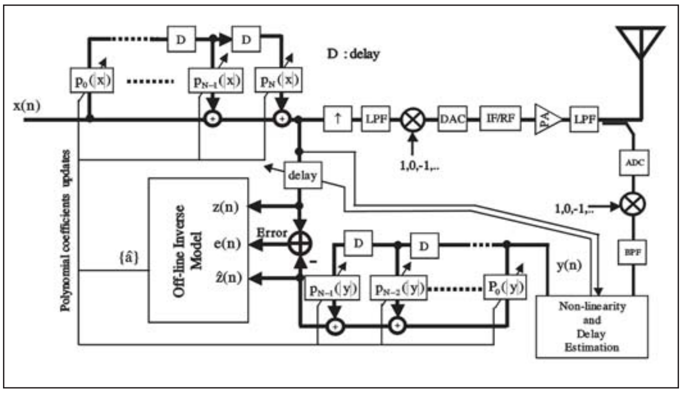

Figure 1 · Hardware implementation of offline inverse PA linearization.

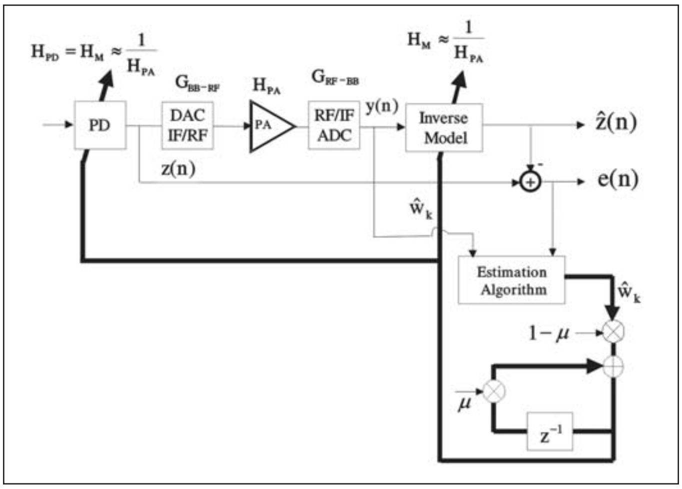

Figure 2 · PA inverse polynomial estimation using offline system identification.

real time. Thus, the offline computation can be carried out in blocks of samples over short durations that are distanced in a long interval (typically minutes), which makes this approach very attractive, expensive, and upgradable.

To illustrate the principle of linearization based on the system in Figure 1 mathematically, consider the mathematical equivalent model illustrated in Figure 2. Here the online pre-distorter (running in real time) and the PA offline model (running at predetermined intervals using only short block of samples) are implemented using eqs. (1) and (2). The task is then for the off-line model to iteratively minimize the error signal given by $z(n) - \hat{z}(n)$ where $\hat{z}(n)$ is the offline estimate of the pre-distorter actual output given by,

$$\hat{z}(n) = x(n)(G_{PD}H_{PD})(G_{PA}H_{PA})(G_{BB-RF}G_{RF-BB})(G_M H_M) \quad (3)$$

where G indicates constant linear gain,

$$H_M = H_M\left(\left|x(n)\right|\right)$$

and

$$H_{PA} = H_{PA}\left(\left|y(n)\right|\right)$$

indicates the discrete non-linear function operating on the input and output of the PA respectively, and n is the sample number, where the actual reference signal $z(n)$ in (3) is given by,

$$z(n) = x(n)(G_{PD}H_{PD})$$

The iterative on-line coefficients identification algorithm would then converge when $z = \hat{z}(n)$, resulting in

$$G_{RF-BB}G_{PA}G_M H_{PA}H_M = 1 \quad (4)$$

If the overall baseband gain in (4) is designed to be equal to the gain of the PA, that is $G_{BB-RF}G_{RF-BB} = 1/G_{PA}$, the off-line PA model would then approximate the PA actual inverse, that is (4) would yield

$$H_{PA}^{-1}\left(|y|\right) = G_M H_M\left(|x|\right)$$

Adaptive Coefficients Estimation

As detailed earlier, the off-line algorithm illustrated in Figure 1 and Figure 2 are obtained iteratively using blocks of captured baseband input and outputs of the PA. There are a number of methods that can be used for the iterative weights estimate of a_{mk} in (2), however, least-square estimation algorithms are preferred for their performance and reasonable complexity. The off-line coefficients in Figure 1, $\mathbf{w} = \{a_{mk}\}$, are first initialized to all zeros except the linear terms of (2), then the inverse required estimator output given by $\hat{z}(n) = \mathbf{w}^T y(n)$ obtained, where T is the transpose of a complex matrix. The modelling error $z(n) - \hat{z}(n)$ is then extracted and used to solve for the next set of weights of (2). This error measure is given by,

$$E\left\{e^2(n)\right\} = E\left\{\left(z(n) - \mathbf{w}^T y(n)\right)^2\right\} \quad (5)$$

Expanding (5), further yields,

$$E\left\{e^2(n)\right\} = E\left\{z^2(n)\right\} - 2\mathbf{w}^T E\left\{z(n)y(n)\right\} + \mathbf{w}^T E\left\{y(n)y^T(n)\right\}\mathbf{w} \quad (6)$$

By inspection of (6), it is revealed that the MSE contains both PA input-output signal cross-correlation matrix **P** and the PA output signal autocorrelation matrix **R**.

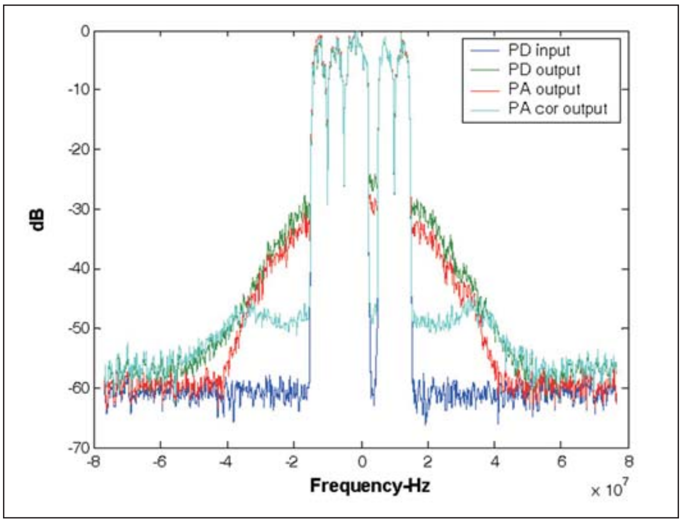

Figure 3 · Simulated illustration of linearization performance using RLS estimator.

Borrowing terminology from [6], the MSE is then given by,

$$E\left\{e^2(n)\right\} = \sigma_d^2 - 2\mathbf{w}^T\mathbf{P} + \mathbf{w}^T\mathbf{R}\mathbf{w} \qquad (6a)$$

The optimal coefficients $\hat{\mathbf{w}}$ are then obtained by setting the derivative of the MSE in (6) with respect to the weights to zero

$$\frac{d}{d\mathbf{w}} E\left\{e^2(n)\right\} = 0$$

which produces the well known Weiner solution given by $\hat{\mathbf{w}} = \mathbf{R}^{-1}\mathbf{P}$. Clearly, to obtain this a large complex matrix inversion has to take a place $\mathbf{R}^{-1}$ that has dimensions of KL, where L is the memory depth, and K is the polynomial order of (2). Fortunately, several iterative algorithms that have lower complexity and do not involve inverting a matrix exists and are available, such as the least mean square (LMS) and recur-sive least square (RLS) [6]. There are also a number of other approaches to obtaining the weights iteratively [7, 8, 9]; among these are the exploitation of neural nets and genetic algorithms, the use of Volterra series, and the use of Hammerstein system. After investigating these algorithms, it was determined that the RLS algorithm seems to be the preferred method due to its accuracy, stability, and fast convergence. The LMS algorithm has very slow converging and noisy weight estimates, but in some scenarios it was able to sustain some disturbances that the RLS could not cope with. This was mainly traced to the instances, where the PA input signal was no longer considered stationary.

Simulated Performance

Using computer simulations, a RLS-based algorithm im-plementation as illustrated in Figure 3 was first simulated with the PA input being three WCDMA-like channels as illustrated in Figure 2. The offline model and online pre-distorter were implemented using (1) and (2) respectively with three taps (L = 3) and a 7th order polynomial (K = 7). Figure 3 illustrates the spectrum of the PA and pre-distorter signals before and after linearization after 8000 iterations using a sampling frequency of 153 Msps. The figures shows that correction is in the order of ~27 dB, which was a very optimistic figure, but explainable when considering that the forward and reverse paths were ideal with perfect timing alignment, high transmitter SNR, and the algorithm was implemented using floating point mathematics.

Experimental Performance

The system in Figure 1 was implanted to linearize a

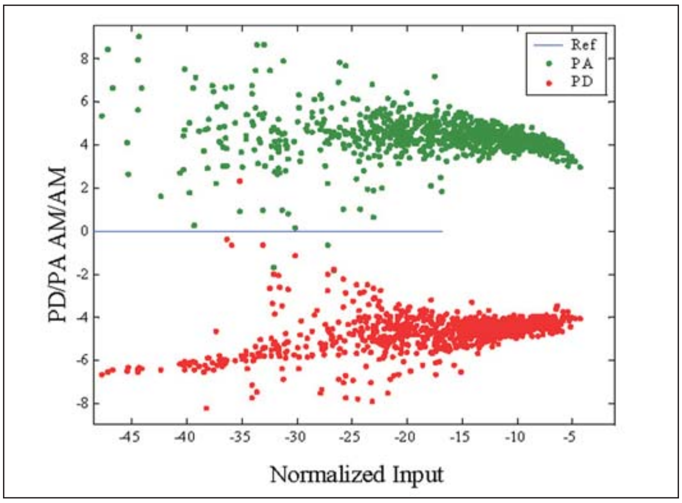

Figure 4 · The measured PD/PA AM/AM transfer functions.

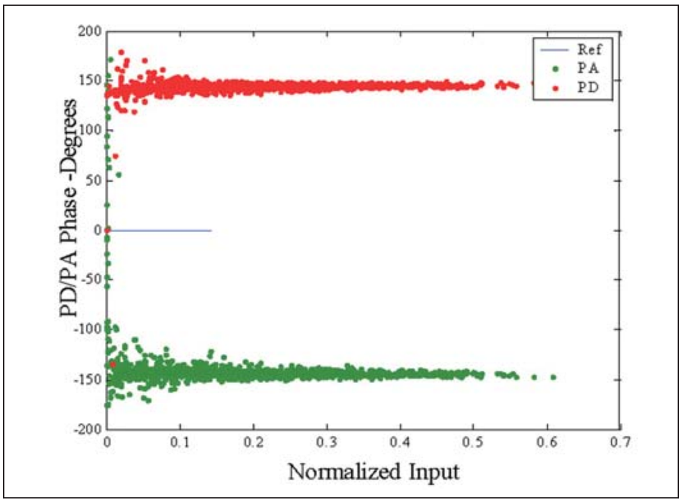

Figure 5 · The measured PD/PA AM/PM transfer functions.

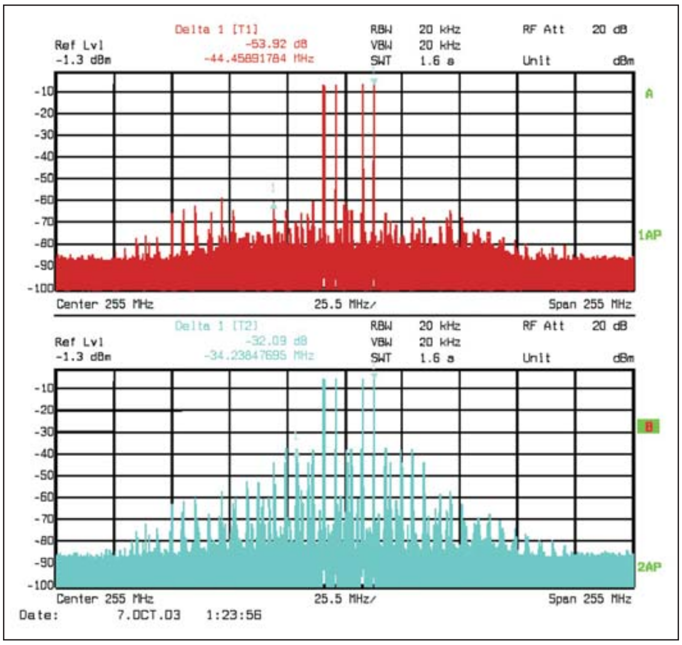

Figure 6 · Measured overall linearization performance.

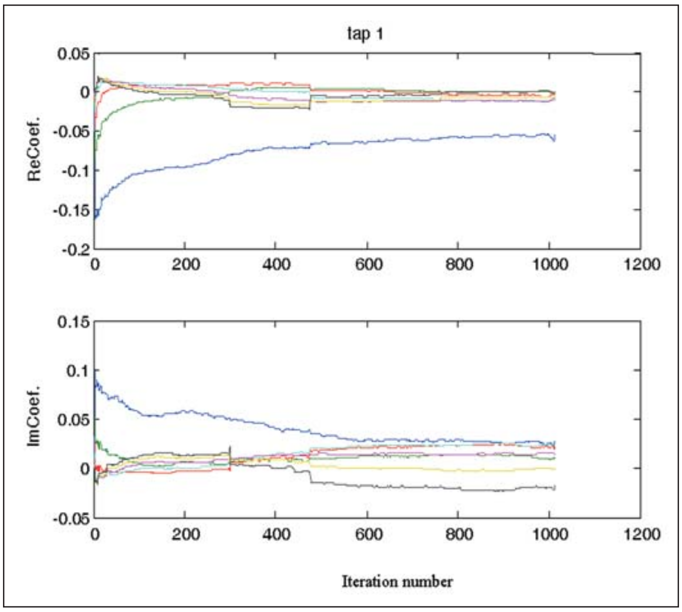

Figure 7 · Measured coefficients of the first pre-distorter tap.

power amplifier with a moderate memory and an average output power of 41 dBm. Figure 4 and Figure 5 illustrate the measured complex transfer functions of the PA and PD captured in the digital domain before and after linearization, respectively. It is observed how the transfer function of the PD is an approximate inverse of the PA transfer function in both figures.

Figure 6 shows the measured spectrum of the PA output signal before and after linearization, where the correction was close to 22 dB over the bandwidth of 10 MHz. Figure 7 shows a snap shot of the measured RLS algorithm adaptive weight estimates for the first tap of (2). It is observed that the algorithm converges in less than 1000 samples, where the sampling rate was 150 Msps.

One way to validate the RLS weights estimates in Figure 2 during various iterations is to observe the signals of $\hat{z}(n)$, $z(n)$, and the mean square error signal as illustrated in Figure 8. Here the MSE (top graph) is computed as

$$20\log_{10}(|z(n) - \hat{z}(n)| / |z(n)|)$$

while the bottom figure shows the estimated versus actual pre-distorter signal, in-dicating very rapid convergence for the case of the RLS with forgetting factor of 1 [6].

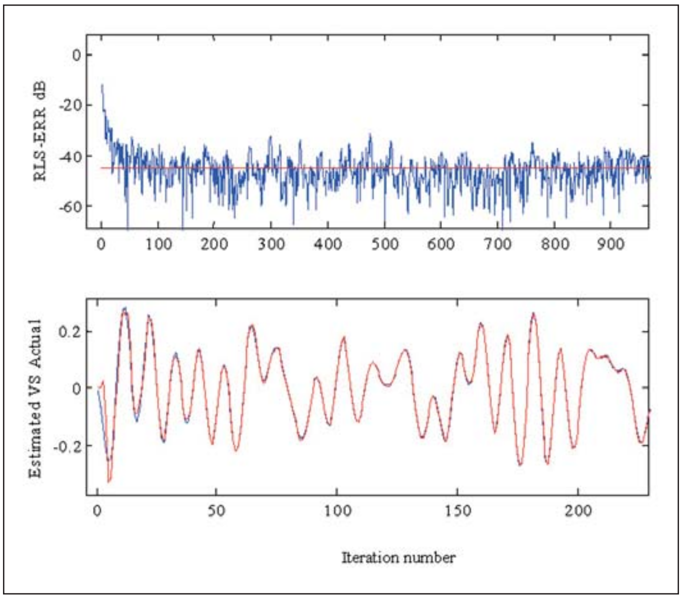

Figure 8 · The adaptive algorithm error signal (top), and a comparison between the actual and the off-line estimated PD output.

Conclusion

Using theoretical analysis, computer simulations, and hardware prototyping, this paper has illustrated the use of inverse modeling in identifying and linearizing a non-memory-less RF amplifier. It is evident from the experimental results, that linearization improvement of more than 15 dB can be obtained over a bandwidth that is suitable for third and fourth generation broadband wireless services.

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About the Author

Dr. Nezami has taught as an adjunct professor and worked as a researcher and developer of electronic systems in USA for 15 years. Until 2004, Dr. Nezami was a Sr. Principal Design Engineer with the Raytheon Company in St. Petersburg Florida-USA, where he mainly worked on digital ground radios and satellite transponder designs. He is currently teaching at the Princess Sumaya University for Technology, a non-profit university in Amman Jordan. Dr. Nezami has a Ph.D. from Florida Atlantic University (2001) and both M.S. and B.S. in Electrical Engineering from the University of Colorado (1988 and 1991). Dr. Nezami has numerous electrical engineering journal and conference publications, and holds three USA patents. He is a USA ham radio operator with the call-sign of KI4CUA.