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Notes on Shannon's Theorem

Given a discrete memoryless channel (meaning that each signal symbol is perturbed by noise independently of the noise effects on all other symbols) with capacity C bits per second, and an information source with rate R bits per second where R < C, there exists a code such that the output of the source can be transmitted over the channel with arbitrarily small probability of error.

—The definition of Shannon's theorem, as described in [1].

In developing his theorem, Claude Shannon effectively separated the transmitted signal from the information being carried. Note that it says nothing about bandwidth or filtering of the signal, or the complexity of the code. And although it says you can establish near-errorless communications, it says nothing about the net data rate (throughput) of that communications after coding is applied.

As described in [2], "What Shannon says here is that in a noise channel, errorless communication (not errorless transmission!) can occur as long as two conditions are met: first, that the information rate R is below a certain value C; and second, that a sufficiently capable code is being used."

The type of idealized assumption made by Shannon to create a mathematical basis for his theorem is common, but there are always practical factors for achieving system performance that approaches the ideal limit. This was addressed later, in the well-known Shannon-Hartley equation:

$$C = B\log_2\left(1+\frac{P_S}{P_N}\right) \quad \text{bps} \qquad (1)$$

where C is the theoretical channel capacity in bits per second, B is the idealized channel bandwidth in Hz, P_S is the total signal power (in watts) and P_N is the total noise power (in watts) within bandwidth B. The ratio P_S/P_N is also called the signal-to-noise radio, or SNR.

The above equation applies real-world factors to Shannon's theorem. We now have C defined as a relationship between bandwidth and SNR. But this modification of the C from the ideal does not change the theorem in any way; it only defines the reduction in C in a practical implementation.

Eq. (1) is often modified by moving B (bandwidth) to the left-hand side, where C/B has the dimensions of bps/Hz. Figure 1 shows the rearranged Eq. (1) plotted on a log-log scale. Below 0 dB SNR, the plot is linear; above 0 dB SNR, the plot flattens but continues to increase with increasing SNR.

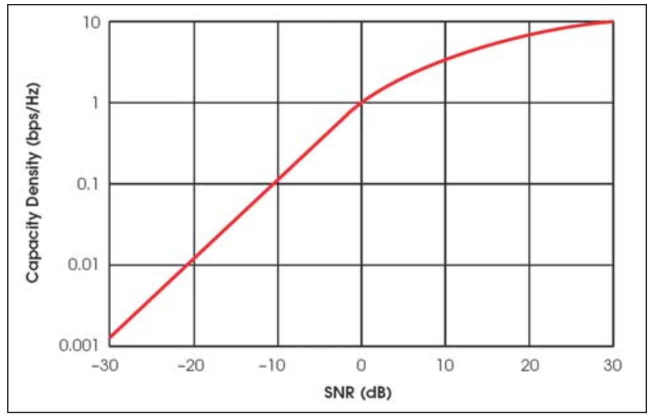

Figure 1 · Plot of capacity density versus SNR.

What this plot shows us is that below 0 dB SNR, where noise is the dominant factor, the capacity of a data channel is reduced in proportion to the SNR (log/log scale). "Below the noise" communications has been used in many applications over the past 30-40 years, confirming Shannon's theorem that such communications is possible, but at a reduced net data rate. These systems now use digital coding, but early systems used equivalent analog methods such as long integration times and ultra narrowband filtering.

In the region at least 6 dB above 0 dB SNR, noise is no longer the limiting factor. In this region, achieving the maximum channel capacity depends on the design of the signal—modulation type and coding. High SNR means that there is little ambiguity in a signal's relative amplitude and phase. Modulation types such as 8 PSK and various levels of QAM contain more bits per symbol, resulting in higher net data rates.

Before Shannon, only the part of Fig. 1 below 0 dB SNR was understood. In those early days of radio (and wireline) communications, all effort for improvement was directed toward achieving a better SNR—higher power transmitters, lower noise figure receivers, higher gain antennas, interference reduction, etc. Shannon's theorem introduced the power of coding, giving engineers a new tool to use for designing improved communication systems. His groundbreaking work has had a dramatic, and lasting impact.

References

1. R. E. Zimmer, W. H. Tranter, *Principles of Modern Communication Systems, Modulation and Noise*, Houghton Mifflin Co., 1976, p. 422.

1. Earl McCune, *Practical Digital Wireless Signals*, Cambridge University Press, 2010, Ch. 2, sec. 2.7, and Ch. 3.