Transmission Line Notes from a Classic Text

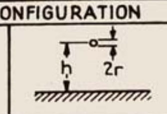
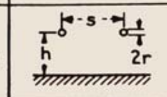
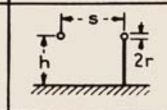
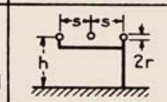
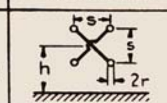
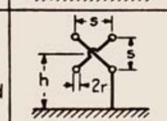
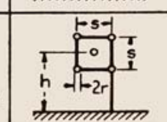
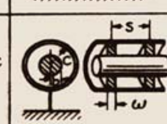
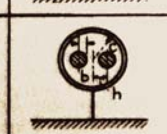
In one of my favorite older textbooks [1], the topic of transmission lines is covered thoroughly, which is one of the reasons why I keep going back to this book when I need to review the basics

The figure reproduced here shows the formulas used to determine the characteristic impedance of several transmission line configurations when run above a perfect ground plane. Although many of these types are no longer used as stand-alone lines such as transmitter-to-antenna feeders, there is useful information for current designs.

If you reduce the scale to millimeters or fractions of an inch, some of these transmission lines resemble multi-layer circuit board systems and metal layers on integrated circuit substrates. With recent techniques in air bridge fabrication techniques for both MMICs and MEMs, the resemblance is even stronger.

For example, the "2-Wire balanced" looks like it's related to coplanar waveguide, and the "3-Wire, 2 wires grounded" might have a corollary with some microstrip configurations. The "4-Wire balanced" and "Shielded pair balanced" resemble some high speed digital interconnecting cables.

Of course, these are all derived from the basic definition:

LOGARITHMS TO THE BASE 10			I_1 = GENERATOR CURRENT
LINE CONFIGURATION		CHARACTERISTIC IMPEDANCE	NET GROUND-RETURN CURRENT
Single wire		$Z_0 = 138 \text{ Log } \frac{2h}{r}$	$I_{Gnd} = I_1$
2-Wire balanced		$Z_0 = 276 \text{ Log } \frac{s}{r}$	$I_{Gnd} = 0$
2-Wire 1 wire grounded		$Z_0 \approx 276 \frac{\text{Log } \frac{s}{r} \text{ Log}\left[\rho^2 \frac{s}{r}\right]}{\text{Log}\left[\rho^2 \left(\frac{s}{r}\right)^2\right]}$ $\rho = \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\text{Log } \frac{s}{r}}{\text{Log } \frac{2h}{r}}$
3-Wire 2 wires grounded		$Z_0 \approx 69 \left[\text{Log } \frac{s^3}{2r^3} - \frac{\left(\text{Log } \frac{s}{2r}\right)^2}{\text{Log } \frac{2h^2}{rs}}\right]$	$I_{Gnd} \approx I_1 \frac{\text{Log } \frac{s}{2r}}{\text{Log } \frac{s\rho^2}{2r}}$ $\rho = \frac{2h}{s}$
4-Wire balanced		$Z_0 = 138\left(\text{Log } \frac{s}{r}\right) - 21$	$I_{Gnd} = 0$
4-Wire 2-wires grounded		$Z_0 \approx 138 \left[\frac{\text{Log } \frac{s}{r\sqrt{2}} \text{ Log}\left[\rho^4 \frac{s}{r\sqrt{2}}\right]}{\text{Log}\left[\rho^4 \left(\frac{s}{r\sqrt{2}}\right)^2\right]}\right]$ $\rho = \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\text{Log } \frac{s}{r\sqrt{2}}}{\text{Log } \frac{\rho^2 s}{r\sqrt{2}}}$
5-Wire 4 wires grounded		$Z_0 \approx 138 \left[\text{Log } \frac{2h}{r} - \frac{\left[\text{Log } 2\rho^2\right]^2}{\text{Log}\left[\rho^3 \frac{h\sqrt{2}}{r}\right]}\right]$ $\rho \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\text{Log } \frac{s}{r4\sqrt{2}}}{\text{Log } \frac{s\rho^4}{r\sqrt{2}}}$
Concentric (coaxial)		$Z_0 = 138 \frac{\text{Log } \frac{c}{b}}{\sqrt{1 + \frac{(\varepsilon - 1)\omega}{s}}}$ ε = Dielectric constant of insulating material	
Double coaxial balanced		$Z_0 = 276 \frac{\text{Log } \frac{c}{b}}{\sqrt{1 + \frac{(\varepsilon - 1)\omega}{s}}}$	
Shielded pair balanced		$Z_0 = \frac{120}{\sqrt{\varepsilon}}\left[2.303 \text{ Log}\left(2v \frac{1-\sigma^2}{1+\sigma^2}\right) - \frac{1+4v^2}{16 v^4}\left(1 - 4\sigma^2\right)\right]$ ε = Dieletric constant of medium ε = Unity for gaseous medium $v = \frac{h}{b}$; $\sigma = \frac{h}{c}$	

Characteristic impedances of various transmission lines over perfectly-conducting earth. (Figure 38 in Ref. 1.)

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{1 + \frac{R}{j\omega L}}{1 + \frac{G}{j\omega C}}}$$

where Z (= $R + j\omega L$) is series impedance per unit length and Y (= $G + j\omega C$) is shunt capacitance per unit length. R, L, C and G are resistance, inductance, capacitance and leakage, respectively, per unit length. It all starts with the basics—resistance and reactance values per unit length of the transmission line in each configuration.

Computing these basic parameters for a given physical structure allows you to determine the characteristic impedance. By adjusting conductor sizes and spacing, to each other and to ground, a transmission line of desired characteristic impedance can be created for your specific application.

Reference

1. F. E. Terman, *Radio Engineer's Handbook*, First Edition, 1943, McGraw-Hill Book Co., Section 3, "Circuit Theory."