L-Network Design Procedure

For maximum power transfer, source and load impedance must have a conjugate match. For many circuits with modest bandwidth requirements, a simple L-network comprising two reactive components is the simplest method of achieving that match. This note is a quick review of L-network design.

Figure 1 shows the sequence of steps for matching two impedances, either of which can be considered “source” or “load.” Fig. 1a shows only the resistance of these impedances; their reactances will be dealt with later. Most often, one of the impedances is a common system impedance. For this example, we’ll assume that \( R_1 = 50 \) ohms, then arbitrarily choose \( R_2 = 10 \) ohms.

The design process begins with a shunt component (\( X_P \) in Fig. 1b) connected in parallel with the higher resistance (\( R_1 \)). The parallel combination will result in a lower resistance, which we want to be equal to \( R_2 \) (10 ohms). Because \( X_P \) is reactive, the resulting lower resistance will now have an associated reactance. This is cancelled by the L-network’s series reactance \( X_S \), which is the negative of the reactance introduced by \( X_P \). Thus, the series and shunt components are of opposite reactances—a shunt inductor and a series capacitor, or vice versa.

The design calculation starts by determining circuit \( Q \) according to the ratio of the two resistances:

\[
Q = \frac{R_p}{R_s} - 1
\]

where \( R_p \) is the resistance adjacent to the parallel leg of the network, and \( R_s \) is the resistance at the series-connected end. \( Q \) may not be negative, so \( R_p \) must be the higher of the two resistances. In our example, \( R_1 = R_p \) and \( R_2 = R_s \). Note that \( Q \) is determined by the source and load, not be user-selected as is typical when designing higher order networks.

The calculation continues as follows:

\[
Q_s = \frac{|X_s|}{R_s} \quad \text{or} \quad |X_s| = Q_s R_s
\]

\[
Q_p = \frac{R_p}{|X_p|} \quad \text{or} \quad |X_p| = \frac{R_p}{Q_p}
\]

where the reactances are given as magnitude only, since \( X_s \) and \( X_p \) may be either capacitance or inductance, but as noted above, cannot both be the same.

Applying these equations to our example, we find that \( Q = 2 \), \( |X_s| = 20 \) ohms and \( |X_p| = 25 \) ohms. Before assigning a sign to each reactance, let’s look at our example with complex impedances at \( Z_1 \) and \( Z_2 \), as in the circuit of Fig. 1c. Since we assumed a system impedance, let \( Z_1 = 50 +j0 \) ohms (same as \( R_1 \)). Then let’s say that \( Z_2 = 10 -j10 \) ohms.

In Fig. 1c we see that \( X_S \) has been split into two parts: \( X_2 \) is the reactive part of \( Z_2 \), and \( X_C \) is the final circuit value that makes \( X_S \) equal to the sum of \( X_C \) and \( X_2 \). We have two choices for the signs of the network reactances, and can examine the effect of each for obtaining practical values of \( X_C \) and \( X_P \).

If \( X_S = +20 \) ohms, then \( X_C \) must be +30 ohms. \( X_P \) will be –25 ohms.

If \( X_S = -20 \) ohms, then \( X_C \) must be –10 ohms. \( X_P \) will be +25 ohms.

One common choice is selecting the configuration to enable or block DC continuity. Let’s say that \( Z_2 \) is a transistor output being matched to a 50 ohm transmission line. Since \( X_C = -10 \) ohms, it is a capacitor and will be useful for blocking DC.

Finally, if \( Z_1 \) is complex, \( X_P \) would be combined with the reactive portion of \( Z_1 \) to compute the actual circuit component value, as was done to find \( X_C \).