From April 2004 *High Frequency Electronics*
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The Magic of Quadrature (I and Q) Signals

Editor:

I am an engineering student, and recently completed a class which introduced the concept of in-phase and quadrature (I & Q) signals. I can work all the math to get the answers to the test problems, but I'd like a better explanation of why this is such a powerful technique. Our professor said that all types of modulation can be described in terms of I & Q, but we didn't have enough examples to see how this is possible. Can you help?

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Some Explanations and an Example

Let's start by confirming your professor's assertion that all modulation types can be represented by a combination of I and Q signals. Intuitively, we can think of each frame of reference as a plane, which is represented in two dimensions—time (inverse of frequency) and amplitude.

Visually, this would make the I and Q channels two flat surfaces that are at right angles to one another. Each is virtually invisible when viewed edge-on from the other. Electrically, this means that the I and Q signal channels are independent, having essentially no effect on the other when the quadrature is an accurate 90 degrees.

At the risk of repeating your class material, we'll review one of the common examples where exact quadrature (90 degree) phase shift has a direct result on the output signal—single sideband suppressed carrier (SSBSC). A block diagram is shown in Figure 1.

To simplify the math, we'll use one carrier frequency f_c and one modulating frequency f_m. As a further simplification, we'll define these signals in the time domain: $f_c = 4\cos(Fc)$ and $f_m = 2\cos(Fm)$, where $Fc = 2\pi f_c t$ and $Fm = 2\pi f_m t$. The constants 4 and 2 are applied to avoid fractions in the math.

As seen in Figure 1, each signal input is split into two parts and applied to different paths. The lower path can be called the I or in-phase channel, while the upper path introduces a 90-degree phase shift to each frequency, represented mathematically by changing from the cosine to the sine function. Each modulator (or mixer) provides a multiplication function, which gives us two equations, one for each modulator output. We'll call them A_I and A_Q:

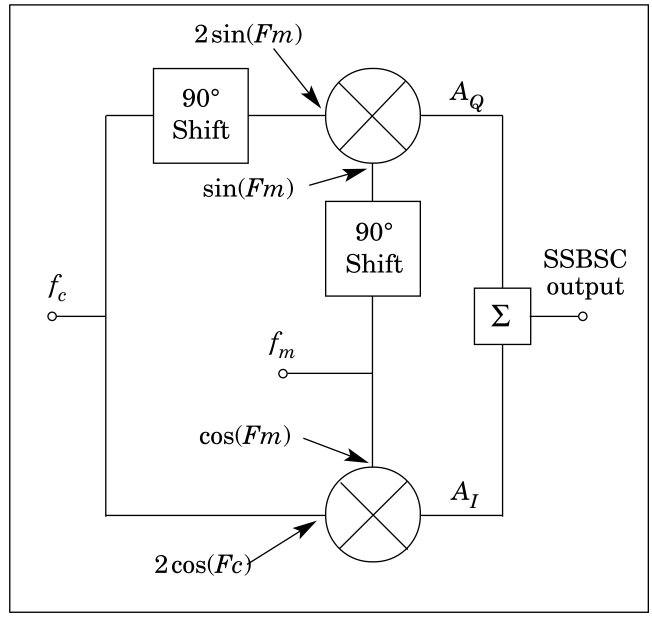

Figure 1 · SSBSC modulation illustrates the use of quadrature (I & Q) signal paths.

$$A_I = 2\cos(Fc) \cdot \cos(Fm)$$

$$A_Q = 2\sin(Fc) \cdot \sin(Fm)$$

and by simple trigonometric identities, we get:

$$A_I = \cos(Fc - Fm) + \cos(Fc + Fm)$$

$$A_Q = \cos(Fc - Fm) - \cos(Fc + Fm)$$

These I and Q channels are then summed to get:

$$(A_I + A_Q) = 2\cos(Fc - Fm)$$

which consists solely of a signal that is the difference between the carrier and modulating frequencies (the lower sideband). Taking the difference of the I and Q channels yields the upper sideband:

$$(A_I - A_Q) = 2\cos(Fc + Fm)$$

Hopefully, this example helps show the relationship between the inputs and the composite, modulated output in an I and Q signal chain. This type of analysis can be used for other modulation types by using the appropriate equations to represent the signals and the modulation functions.